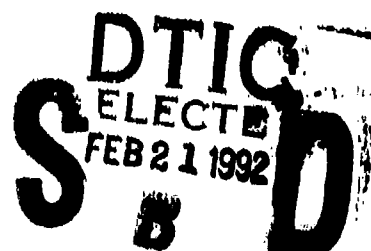


# NAVAL POSTGRADUATE SCHOOL

## Monterey, California

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## THESIS

OPTIMAL MAINTENANCE POLICIES APPLICABLE TO  
REPAIRABLE SYSTEMS ONBOARD SHIPS

by

Sukhdev Singh

March, 1991

Thesis Advisor:

D. P. Gaver

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**Optimal Maintenance Policies Applicable  
to Repairable Systems Onboard Ships**

by

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## ABSTRACT

Maintenance, a key element of Integrated Logistics Support, plays a very vital role throughout an equipment/system planned life-cycle. Maintenance costs contribute a major portion of the life cycle costs of an equipment or system. Past historical records have shown that the cost associated with system maintenance is immense and usually takes up a large portion of the annual operating expenditure. Besides the costs, sound maintenance efforts contribute to better operational availability and reliability of a system. Therefore, the objective is to attain the proper balance of operations between performance and effectiveness, and logistics support, which largely includes maintenance, spares requirements, and the available budget. Adequate maintenance is essential to ensure the effective and economical support of an equipment or system. Therefore there is a need to design optimal maintenance policies to maximize appropriate measures of system effectiveness. These can be either to minimize operational and maintenance costs, to improve overall system reliability or to maximize operational availability.

In this thesis, various maintenance scenarios are examined and the corresponding optimal maintenance actions are planned to take place at intervals chosen so as to maximize an appropriate measures of effectiveness. Preventive maintenance policies are also planned so that the overall reliability of the system is always kept above a specified minimum reliability level, while either keeping the cost per unit time to a minimum or maximizing the operational availability, subject to cost constraints.



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## **THESIS DISCLAIMER**

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.



## **I. INTRODUCTION**

### **A. MAINTENANCE**

Maintenance constitutes a series of actions to be taken to restore or retain an equipment or system in an effective operational state. [Ref. 1].

Maintenance occurs at three levels:

- Organizational Maintenance.
- Intermediate Maintenance.
- Depot Maintenance.

The Organizational Maintenance activity is performed by the ship's staff, for frigates or equivalent, using the onboard spares, tools, test equipment, and the documentation furnished. For smaller ships and patrol craft, Organizational Maintenance is carried out with the assistance of the Squadron Support Teams. Intermediate Maintenance is performed at fleet workshops, while Depot Maintenance is carried out at the naval dockyard or by commercial shipyards and firms.

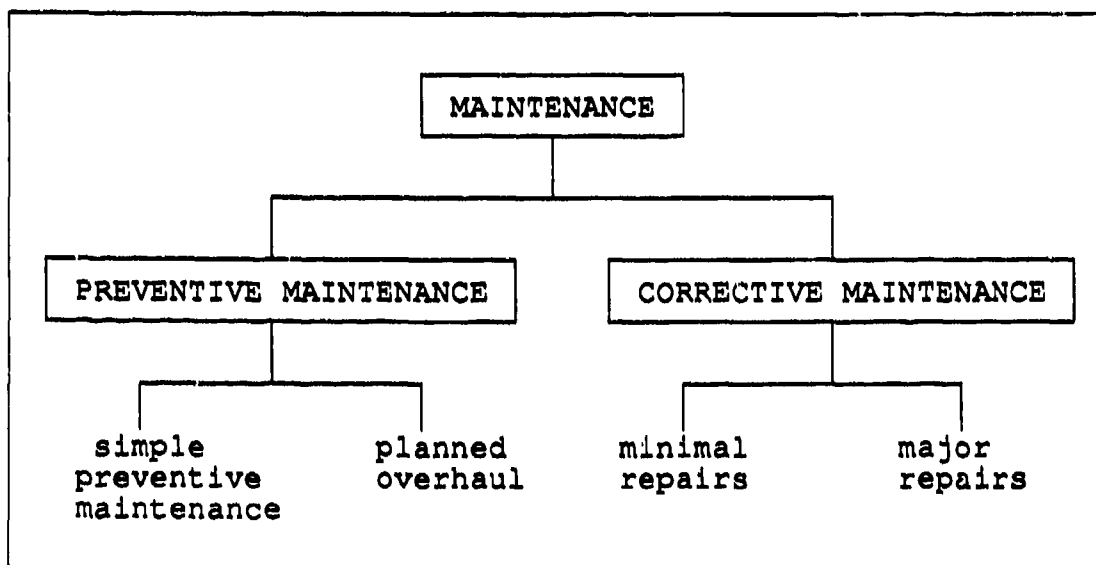
Any equipment or system introduced into the service needs to be maintained adequately so that it is readily available to perform a mission successfully at an acceptable performance level. Besides that, maintenance

helps to extend the useful life of the equipment/system and ensures the safety of personnel using it.

Basically, maintenance can be broadly divided into two types:

- Preventive Maintenance.
- Corrective Maintenance.

Figure 1.1 shows the various maintenance forms.



**Figure 1.1 The Forms of Maintenance**

Preventive maintenance is scheduled maintenance that is organized and carried out in accordance with the documentation and records at a predetermined time following a predetermined plan. Preventive maintenance is normally associated with maintenance that occurs when the system is still

operating. The main aim of preventive maintenance actions is to improve the overall system reliability and to avoid sudden or unexpected failures which may be catastrophic. Preventive maintenance can be sub-divided into:

- Simple Preventive Maintenance.
- Complete Overhaul/Preventive Replacement.

Simple preventive maintenance actions usually consist of inspections, adjustments, tuning, cleaning, lubrication, minor calibration, and replacement of worn out components and parts before they actually fail. This type of maintenance is usually performed at the Organizational Maintenance level and often it does not affect the downtime of the equipment/system.(The downtime for these maintenance actions is usually negligible).

Complete overhauls are carried out to bring the state of the deteriorating system back to "as good as new condition", and to prevent major impending failures. This type of maintenance is usually carried out at the Depot Maintenance level during planned refit periods and it usually incurs some significant downtime. Preventive replacements are carried out when the overhaul is not economical or when the parts are non-repairable.

Corrective maintenance is the unscheduled maintenance carried out to restore a failed system to its operating state. Corrective maintenance can be subdivided into:

- Minimal Repair.
- Major Overhaul/Failure Replacement.

Minimal repairs are minor repairs or component replacements carried out on failed components or assemblies that restore the system to its operational state without significantly improving the overall condition of the system. This type of maintenance is usually carried out at the Organizational Maintenance level.

Major overhauls are carried out when the system experiences a sudden major failure and the work has to be carried out at the Intermediate Maintenance level or at the Depot Maintenance level. Failure replacement usually occurs when the system is beyond economical repair or when the parts are non-repairable.

## **B. MAINTENANCE POLICIES**

Our main goal is to design a maintenance policy that will maximize appropriate measures of system effectiveness; this can be either to minimize operational and maintenance costs, to improve overall system reliability or to maximize operational availability (uptime). The most relevant measure of effectiveness used is to maximize a suitable measure of operational availability subject to budget constraints. Ideally, we would like to carry out many preventive maintenance and inspection routines to ensure that the system is in an optimum operating condition, to avoid sudden and catastrophic failures

and damage to the system, and also to prevent accidents which may be detrimental to the people working in the vicinity. On the other hand, excessive preventive maintenance actions may not only be unnecessarily costly, requiring many manhours but may actually prematurely age equipment. Therefore, we need a balance or trade-off between two extremes. Our objective, then, is to select an optimal maintenance policy for the particular equipment or system, and to decide when to carry out the associated maintenance routines. Some conceivable maintenance policies are the following:

- No preventive maintenance and corrective maintenance is carried out. The system is replaced at a fixed age. When the system fails before this age it is replaced with a new one. This type of maintenance is normally associated with low-level subsystems: items such as components, sealed modules or other non-repairable parts. Some common examples are the magnetron in the radar transmitter unit and the belts found in motors.
- No preventive maintenance is carried out. The system is replaced at a fixed age. For failures that occur in between the planned replacement age, minimal repairs are carried out to restore the system to an operational state. This type of maintenance is usually associated with a system consisting of several components such that when a component fails the system fails, and replacing the component restores the system to operation. Minimal repairs are often carried out in the field, i.e. onboard ships or at a forward air base.
- The system is renewed at a fixed age either by replacement or overhaul after which the system is "as good as new". Failures in between the planned age replacement can be classified into type I and type II. Type I failures are simple failures which are remedied by minimal repairs using the support elements onboard ships, and Type II failures are major failures which require base support facilities and are rectified by part replacement or complete overhaul.
- Both preventive and corrective maintenance actions are employed. Preventive maintenance is planned at some time interval to improve the

Other classes of distributions which exhibit wear out or wear-in such as *NBU* (new better than used) or *UBN* (used better than new) can also be used and are discussed in detail in Barlow and Proschan. [Ref. 2].

Most of the systems which are newly installed onboard the ships initially have a decreasing failure rate, sometimes referred to as the infant mortality phase or running-in period. Then the failure rate becomes rather constant for some time and finally increasing, exhibiting wear out. In reliability, such failure rate functions are said to have a "bathtub" shape. The running-in period is usually under contractual obligation.

#### D. FAILURE RATE

The *failure rate* or *hazard rate* is one of the most important statistical characteristics of any equipment/system frequently used in maintenance or replacement studies.

The failure rate,  $h(t)$ , is usually defined as the "instantaneous" conditional probability of failure at age  $t$ , given that it has survived to age  $t$ . When  $F$  has density  $f$ ,  $h(t)$  is given by

$$h(t) = \frac{f(t)}{1 - F(t)} \quad (1.1)$$

The failure rate  $h(t)$  can also be expressed in terms of the *hazard function*,  $H(t)$

$$h(t) = \frac{dH(t)}{dt} \quad (1.2)$$

The hazard function is related to the distribution function by the following relationship

$$H(t) = -\log \{1 - F(t)\} \quad (1.3)$$

## E. RELIABILITY

When deciding upon system maintenance policies, the frequency of maintenance actions becomes a significant parameter. The frequency of maintenance for a given system is highly dependent on the reliability of that system. In general, as the reliability of a system increases, the frequency of maintenance actions will decrease; conversely, the frequency of maintenance actions will increase as system reliability is degraded. [Ref. 1]

The *reliability function* or *survival function*  $R(t)$  is given by

$$R(t) = 1 - F(t) = \int_t^\infty f(t) dt \quad (1.4)$$

$R(t)$  is the probability that a new system will perform its mission satisfactorily for at least a certain time  $t$ . If  $T$  is the time to failure of the system then

$$R(t) = P(T > t) \quad (1.5)$$

Sometimes we are interested in the chance of survival of the system in the future given that it has survived up till now. This is called the *conditional survival function*. So if  $T$  is the time to failure of the system with the survival

function  $[1 - F(t)]$ , then the reliability of the system,  $R(t;a)$ , at some age  $a \geq 0$  is given by

$$R(t;a) = \frac{[1 - F(t+a)]}{[1 - F(a)]} \quad (1.6)$$

## F. AVAILABILITY

Availability is a measure of system readiness and it is one of the most important measures of effectiveness usually employed in mission-oriented situations especially in the military environment. *Operational availability* is the probability that a system or equipment, when used under stated conditions in an actual operational environment, will operate satisfactorily when called upon [Ref 1]. System availability is influenced both by the inherent failure-proneness of the system and by the time and resources (support elements) it takes to restore a failed system to service. [Ref. 3]. Times to failure or 'up times' and to restoration or 'down times' may vary considerably, and not necessarily independently, depending upon the mode of failure, the time required to diagnose the failure, availability of special tools, test equipment, and spare parts, and the proper documentation and the required personnel skills. The long-run availability or steady state is expressed as follows:

$$A_o = \frac{E[U]}{E[U] + E[D]} \quad (1.7)$$



where

$E[U]$  is the expected uptime of the system

$E[D]$  is the expected downtime of the system

For a system operating at sea the availability at equation (1.6) is usually expressed as

$$A_s = \frac{MTBM}{MTBM + MDT} \quad (1.8)$$

where

MTBM is the mean time between maintenance

MDT is the mean downtime, which includes the mean active maintenance time (M), expected logistics delay time (LDT) and the expected administrative delay time (ADT). The mean active maintenance time includes the expected time for preventive and corrective maintenance.

## G. MAINTENANCE TIME DISTRIBUTION

The time required to carry out *simple preventive maintenance* actions and overhauls can generally be modeled as normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . Most of these tasks are standard and are carried out in accordance with the planned maintenance schedules; which stipulates the procedures to follow, spares, material, tools and test equipment that are required to perform the maintenance actions. [Ref 4]. The tasks usually requires a fixed amount of time to accomplish with very little variation.

The time required for *corrective maintenance* actions can be divided into three basic categories:

- Active repair time.
- Logistics delay.
- Administrative delay.

### **1. Active Repair Time**

Active repair time depends on the environment, state of equipment (hot or cold), and skill level of the technician; and it can be sub-divided into the following categories:

- Recognition or detection ( often the time until actual occurrence of a failure and its recognition is not known).
- Localization or diagnosis.
- Correction or repair.
- Verification or check.

### **2. Logistics Delay Time**

Logistics delay time constitutes downtime that is expended while waiting for the availability of a spare part, waiting for a special tool or test equipment to perform repair, waiting for transportation, and waiting to use a facility required for the repair.

### 3. Administrative Delay Time

Administrative delay time constitutes downtime of administrative nature, such as personnel assignment priority and organizational constraint.

### 4. Distribution

The distributions most commonly used to describe the downtime for corrective maintenance actions are exponential and log-normal. The exponential distribution tends to fit the type of equipment that requires relatively short durations of repair and usually corresponds to the replacement of a failed unit. Occasionally, much longer times may be required for major repair or for spares. The lognormal distribution is useful for situations where there are few downtimes of short duration, a large number of downtimes closely grouped about some modal value and a few downtimes of long durations.

If  $X$ , the downtime, is a random variable having the lognormal distribution given by the probability density function

$$f_X(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \quad 0 \leq x < \infty \quad (1.9)$$

then the logarithm of  $X$  is Normal with mean  $\mu$  and variance  $\sigma^2$ . In the thesis We shall study several maintenance policies applicable to systems onboard ships. In Chapter II we introduce optimal maintenance for three types of policies. These are considered in detail in Chapters III, IV and V.

## **II. OPTIMAL MAINTENANCE POLICIES AND MATHEMATICAL MODELS**

In the commercial environment determination of the optimum maintenance policy and time is of great economic importance. However, in many military situations, failure of a system in an operational environment is not only going to be more costly, but dangerous and may jeopardise the success of a mission. If a system has an increasing failure rate, such as the failure rate of a Weibull distribution with shape parameter  $> 1$ , it may be wise to replace or overhaul the system before it has aged too greatly. [Ref. 5]. This is very true for systems onboard the ships especially when they are operating many hundreds of miles from their home base and hence the support elements are not completely and readily available. A failure at sea may be catastrophic, in terms of cost and operational requirements. Although we can not completely avoid failures, however we can reduce the chance or probability of such catastrophic failures. This can be done by studying the failure distributions and then employing appropriate maintenance actions to maximize the various measures of effectiveness.

The appropriate maintenance actions could be either preventive maintenance, failure replacement, minimal repair and preventive replacement or complete overhaul. The objective of a maintenance policy is to find a

sequence of times for carrying out the various maintenance actions that maximizes the appropriate measures of effectiveness over the operational and maintenance cycle of the system.

The most commonly used maintenance policy is the policy based on age, usually referred as the *age replacement policy*. Sometimes the maintenance policy is based on the running hours of the system or equipment. If a system consists of many identical components, then the maintenance of these components are done in a block or group and is called the *block replacement policy*, such as the replacement of the diodes in the exciter unit of an alternator where the accessibility is poor.

A recent survey of Preventive Maintenance Models for stochastically deteriorating single-unit system [Ref. 6] highlighted the use of some optimization models for repair and replacement policy evaluation. Most of these models were based on minimizing the long-run expected costs per unit time of replacement and minimal repair as the measure of effectiveness. The basic minimal repair model developed by Barlow and Hunter [Ref. 7] has been generalised and modified by many authors to fit more realistic situations.

Minimal repair models generally assume the following: [Ref. 6].

- The system's failure rate function is increasing.
- Minimal repairs do not affect the failure rate of the system.

- The cost of a minimal repair  $C_r$  is less than the cost of replacing the entire system  $C_p$ .
- System failures are immediately detected.

The long-run expected cost per unit time using a replacement age  $t$  for the basic model is given by

$$C(t) = \frac{C_r N(t) + C_p}{t} \quad (2.1)$$

where  $N(t)$  represents the expected number of failures (minimal repairs) during the period  $(0, t]$ .

Using the basic minimal repair model as developed by Barlow and Hunter [Ref. 7], Tilquin and Clérout [Ref. 8] investigated an optimal replacement policy for the case where an adjustment cost  $C_a(ik)$ , incurred at age  $ik$ ,  $i = 1, 2, 3, \dots$  and  $k > 0$ , is added to the basic costs  $C_p$  and  $C_r$ . They showed that the long-run expected cost per unit time is given by

$$C(t) = \frac{C_r N(t) + C_p + C^*_a(v(t))}{t} \quad (2.2)$$

where

$$C^*_a(v(t)) = \sum_{i=0}^{v(t)} C_a(ik)$$

and  $v(t)$  represents the number of adjustments in the period  $(0, t]$ . Tilquin and Clérout [Ref. 8] showed that the global minimum for equation (2.2) exists in the interval  $[0, \infty]$  when the life distribution is IFR.

In the thesis we will examine the various maintenance scenarios here called the policies and then based on these we will formulate the appropriate mathematical models using the stochastic and reliability theory. This mathematical models will depend on the desired measures of effectiveness required. From these models we can obtain the times for carrying out the appropriate maintenance actions. The measure of effectiveness that will be considered are:

- Minimizing the costs.
- Maximizing availability.
- Mission reliability.

The maintenance actions will be different for various measures of effectiveness and it is up to the Decision Maker to select which one is suitable for his scenario.

The following maintenance policies are of interest:

- Policy I    Age Replacement.
- Policy II   Minimal Repair with Age Replacement.
- Policy III   Minimal Repair, Failure Replacement /  
                  Overhaul, Preventive Replacement /  
                  Overhaul.

These policies are discussed in the following chapters.

### **III. POLICY I (AGE REPLACEMENT)**

The system is replaced at the time of failure or at some fixed time  $t_p$ , whichever comes first. (Instead of replacement, we could also overhaul the system, which on completion of overhaul is assumed as "good as new").

This type of scenario is usually associated with the repair by replacement policy often adopted at sea and applies to modules and sub-assemblies of equipment which requires support elements not available at sea. In lieu of repairing these modules and sub-assemblies at sea, sufficient spares are carried onboard or prepositioned at the forward operating areas so as to accomplish the respective missions successfully. The optimal number of spare requirements are based on the measures of effectiveness desired.

The age at which the operating system is replaced depends on the following factors:

- Failure distribution.
- Costs of failure and preventive replacement.
- Downtime of failure and preventive replacement.
- Measure of effectiveness:
  - Minimize costs.
  - Maximize availability.
  - Mission reliability.



When evaluating this policy the following assumptions are made:

- Planned replacements are less costly than failure replacements.
- The mean downtime for a planned replacement is less than that for a failure replacement. An unexpected failure may incur additional Logistics Delay Time (LDT) and Administrative Delay Time (ADT), especially if the failures occurred whilst the ship is at sea.
- The system exhibits an increasing failure rate distribution i.e.  $h(t)$  increases as  $t$  increases.
- The cost and downtime associated with simple preventive maintenance actions and minimal repairs is negligible.
- Preventive maintenance actions and minimal repairs do not improve the reliability of the system.

Let  $T$  be the time to failure of the system.  $T$  is a random variable with an IFR distribution function  $F(t)$ . A cycle is completed everytime a replacement is made. It can be either a failure replacement or a preventive replacement. (The system probabilistically starts over again and each replacement constitutes a renewal). By using the Renewal Reward Process, Ross [Ref. 9]: Expected long run average cost,  $C(t_p)$ , is given by

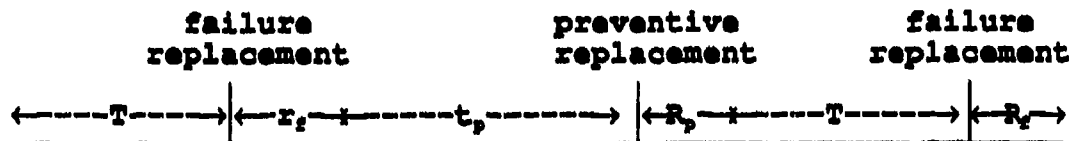
$$\begin{aligned}
 C(t_p) &= \frac{\text{Expected cost incurred during a cycle}}{\text{Expected length of cycle}} \\
 &= \frac{E[C]}{E[L]} \quad (3.1)
 \end{aligned}$$

where

$C$  is the total maintenance cost incurred during a cycle

$L$  is the length of a cycle

Let  $t_p$  be the planned replacement age



$$C = \begin{cases} C_p & T > t_p \\ C_f & T \leq t_p \end{cases} \quad (3.2)$$

where  $C_p$  is the cost of preventive replacement

$C_f$  is the cost of failure replacement

$$C_f > C_p$$

$$L = \begin{cases} R_p + t_p & T > t_p \\ R_f + T & T \leq t_p \end{cases} \quad (3.3)$$

where  $R_p$  is the time of preventive replacement

$R_f$  is the time of failure replacement

$$R_f > R_p$$

Then,

$$C(t_p) = \frac{C_p[1-F(t_p)] + C_f[F(t_p)]}{[R_p + t_p][1-F(t_p)] + R_f F(t_p) + \int_0^{t_p} t f(t) dt} \quad (3.4)$$

and the above equation can be simplified to the standard form as shown in Barlow [Ref. 7] and Jardine [Ref. 10]. See Appendix B for details.

$$C(t_p) = \frac{C_p[1-F(t_p)] + C_f[F(t_p)]}{R_p[1-F(t_p)] + R_f F(t_p) + M(t_p)} \quad (3.5)$$

where  $M(t_p)$  is the mean life during a cycle and is given by

$$M(t_p) = \int_0^{t_p} [1-F(t)] dt \quad (3.6)$$

The optimal preventive replacement age can be found by finding the value of  $t_p$  that minimizes the cost function in equation (3.4) and the optimal value of  $t_p^*$  is that value that satisfies the following equation [Appendix B]

$$h(t_p) \int_0^{t_p} [1-F(t)] dt - F(t_p) = \frac{C_p}{C_f - C_p} + h(t_p) \left[ \frac{R_f C_p - R_p C_f}{C_f - C_p} \right] \quad (3.7)$$

The other measure of effectiveness that the Decision Maker is often interested is the *availability* (the probability the system is up at any time  $t$ ).

Assuming that there are only two states, that is the system is either up or down, then from the Regenerative Process Ross [Ref 9], Availability,  $A(t)$ , is the Expected amount of time the system is up during a cycle divided by the Expected time of a cycle and is given by

$$A(t) = \frac{\text{Mean life during a cycle}}{\text{Expected length of a cycle}}$$

$$A(t_p) = \frac{M(t_p)}{R_p[1-F(t_p)] + R_p F(t_p) + M(t_p)} \quad (3.8)$$

The optimal preventive replacement age can be found by finding the value of  $t_p$  that maximizes the availability function in equation (3.8) and the optimal value of  $t_p^*$  is that value that satisfies the following equation. [Appendix B]

$$M(t_p) = \left[ \frac{R_p}{(R_f - R_p)} + F(t_p) \right] \frac{1}{h(t_p)} \quad (3.9)$$

The other measure of effectiveness of interest is the *mission reliability* which is defined to be the probability that the system will complete a certain mission of duration  $d$  when it is at age  $t$ . This is given as follows

$$\begin{aligned} R(t, d) &= \frac{R(t + d)}{R(t)} \\ &= \frac{1 - F(t + d)}{1 - F(t)} \end{aligned} \quad (3.10)$$

In military applications we usually like to maximize the availability of the system subject to some budget constraint  $C$ :

$$\begin{aligned} \max \quad & A(t_p) \\ \text{s.t.} \quad & C(t_p) \leq C \\ & t_p > 0 \end{aligned} \quad (3.11)$$

Some of the continuous life distributions that are commonly used to model the increasing failure rate of the system are:

- a. Weibull.
- b. Gamma with shape parameter  $> 1$ .
- c. Log-Normal (depends on parameter).

If we do not know the probability life distribution then we have to resort to a nonparametric approach [Ref. 10].

It should be noted that if the failure rate of the system is constant i.e. it exhibits the Exponential distribution then by virtue of its memoryless property we do not carry out any preventive replacement no matter what because a new system is just as bad and good as an old one.

For our case let us assume that from historical data we know that this system has a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$ . Then the probability distribution function  $F(t)$  is given by

$$F(t) = 1 - e^{-(\lambda t)^\alpha} \quad \alpha > 1, \lambda > 0, t \geq 0 \quad (3.12)$$

and the failure rate  $h(t)$  is given by

$$h(t) = \alpha \lambda t^{\alpha-1} \quad \alpha > 1, \lambda > 0, t \geq 0 \quad (3.13)$$

and the hazard function  $H(t)$  is given by

$$H(t) = (\lambda t)^\alpha \quad \alpha > 1, \lambda > 0, t \geq 0 \quad (3.14)$$

It should be noted that if the shape parameter  $\alpha = 1.0$ , then the failure rate,  $h(t) = \lambda$ , which is a constant and this corresponds to an Exponential distribution whose distribution function is given by

$$F(t) = 1 - e^{-(\lambda t)} \quad \lambda > 0, t \geq 0 \quad (3.15)$$

Figures 3.1 and 3.2 shows the plot of the Weibull density function and the corresponding hazard function respectively. It can be observed that as  $\alpha$  increases the failure rate increases more rapidly.

It should be noted that for a Weibull distribution

$$M(t_p) = \int_0^{t_p} e^{-(\lambda t)^\alpha} dt \quad (3.16)$$

has no simple elementary closed form solution, but can be expressed in terms of the incomplete Gamma function.

## A. NUMERICAL ILLUSTRATION

A case example was taken with the parameters as follow:

$$C_p = \$ 25000$$

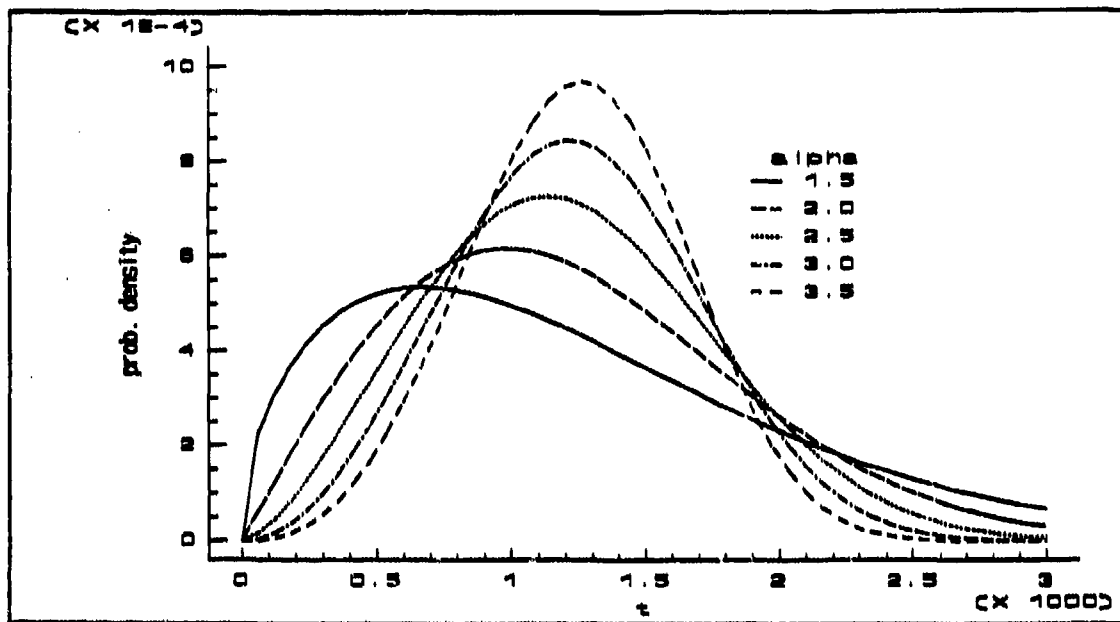
$$C_r = \$ 37500$$

$$R_p = 8 \text{ hours}$$

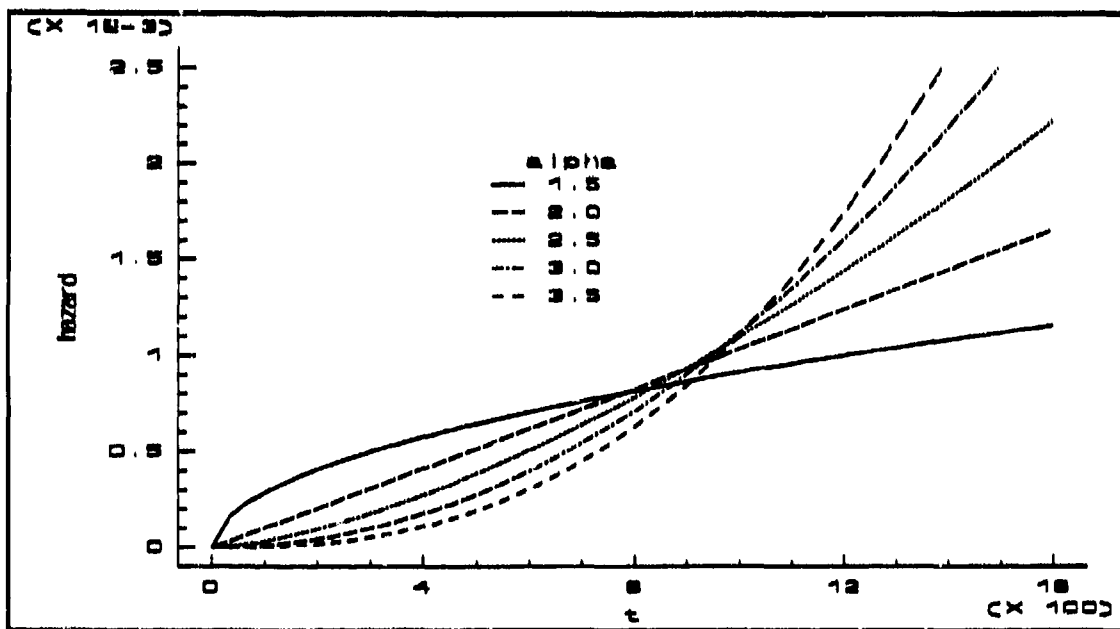
$$R_r = 16 \text{ hours}$$

$$\alpha = 3.0$$

$$\lambda = 1/1390 \text{ hours}$$



**Figure 3.1 Weibull Density Function  $f(t)$  for  $\alpha > 1$ , and  $1/\lambda = 1390$  hours**



**Figure 3.2 Hazard Function of the Weibull distribution  $H(t)$ ,  $\alpha > 1$  and  $1/\lambda = 1390$  hours**

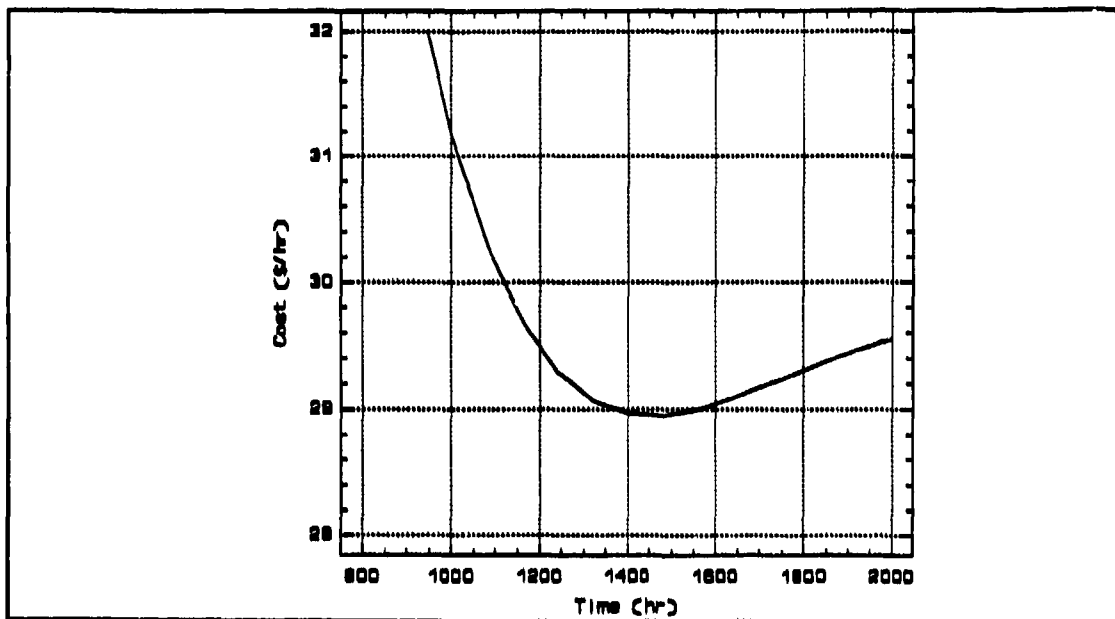
The above example was extracted from [Ref.12]. This example was based on the overhaul of aircraft engines. This same principle can be applied to the overhaul of motors, compressors pumps and weapon systems onboard ships.

An *IMSL* subroutine *DQDAG* was available on the IBM3033P main frame computer at the Naval Postgraduate School, [Ref. 13], which could integrate the function very accurately and expeditiously by using a globally adaptive scheme based on *Gauss-Kronrod* rules. The estimate of the absolute value of the error using this scheme was  $10^{-11}$ . A brief description of this subroutine is shown in Appendix C.

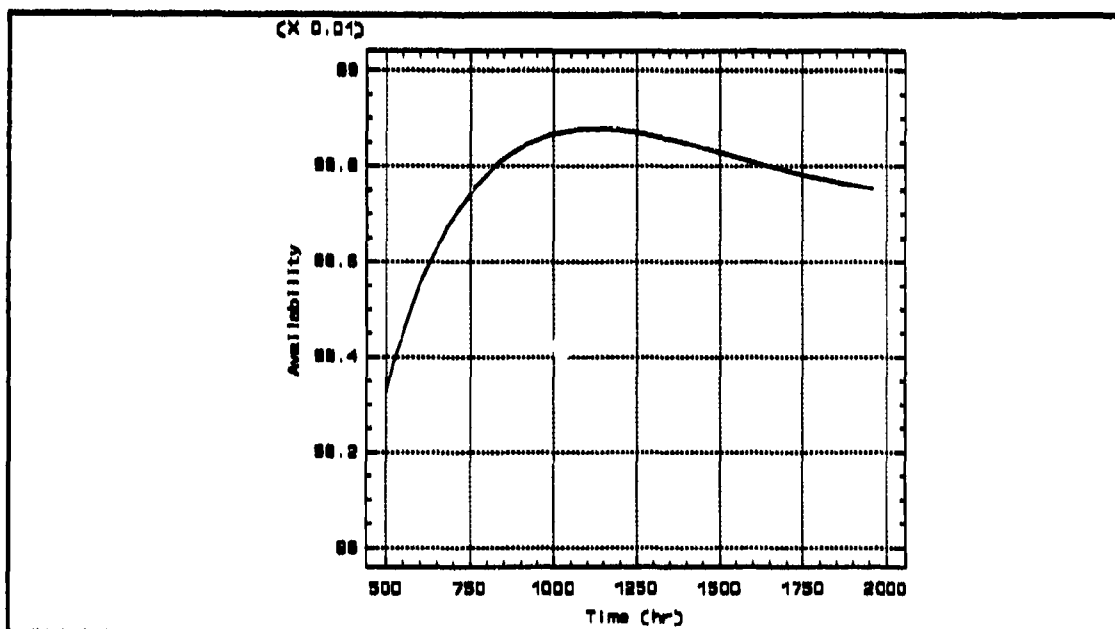
The cost function and the availability function as in equations (3.5) and (3.8) respectively were plotted against time. Figures 3.3 and 3.4 shows these plots respectively. It is observed that the cost is minimum at about  $t_p^* = 1453$  hours and at a rate of about \$28.95 per hour. At this critical value of  $t_p$  (1453 hours) the availability is about 0.9883. It is also observed that the availability is maximum at  $t_p^* = 1126$  hours giving an availability of 0.9888 . For this maximum value of availability we need a budget of at least \$29.92 per hour. Based on this information it is up to the Decision Maker to choose the optimal time  $t_p^*$  to carry out preventive replacement or overhaul.

From the graphs in Figures 3.3 and 3.4 it is observed that the plots are fairly flat and this gives some flexibility to the Decision Maker. The replacement age need not occur exactly at the optimal age.





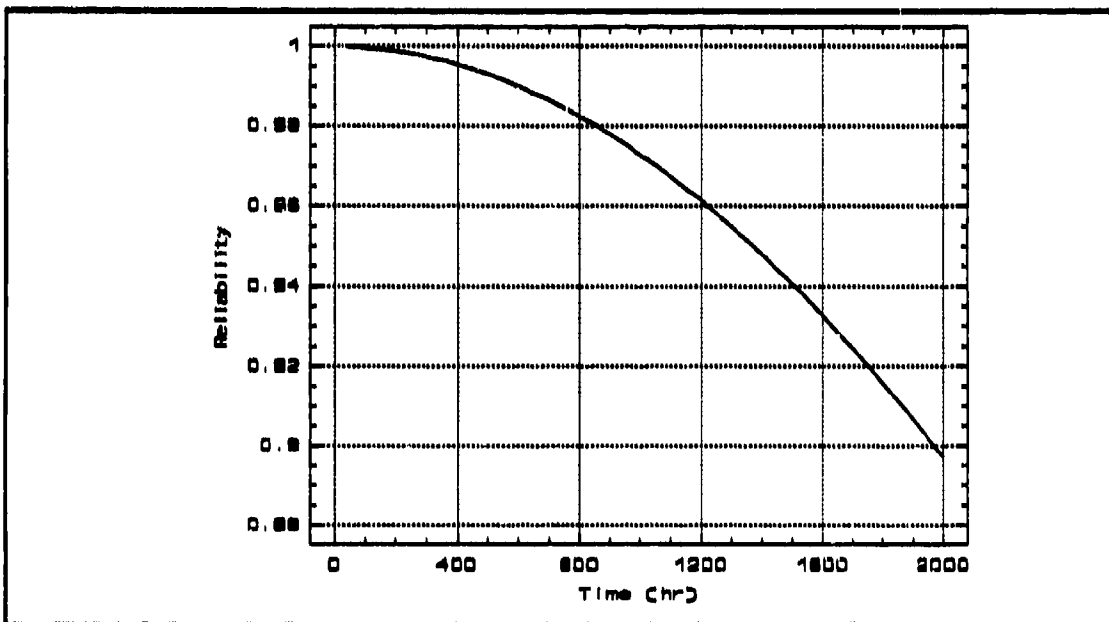
**Figure 8.8 Plot of the Cost Rate Function,  $C(t_p)$ .**



**Figure 8.4 Plot of the Availability Function,  $A(t_p)$ .**

The optimal values of  $t_p^*$  for the minimum cost and the maximum availability as mentioned above also satisfies the equations (3.7) and (3.9) respectively. The values are 1453.45 and 1126.38 hours respectively.

The Decision Maker may also be interested in the mission reliability of the system / equipment. The graph in Figure 3.5 shows the probability that the system / equipment will sustain 24 hours of continuous operation successfully when the equipment is at a certain age, say  $t$  hours. Suppose the Decision Maker wants a reliability of not less than 0.95; then the optimum replacement age  $t_p^*$  is 1371 hours.



**Figure 3.5 Plot of the Reliability Function,  $R(t)$ .**

From the plot in Figure 3.5 it is observed that the reliability decreases as the equipment ages. This shows that even if the equipment does not fail and

if it has to sustain an operation for a specified duration of time, an older equipment will be less reliable and thus may not fulfill an operational mission successfully.

Table 3.1 shows the values of the cost rate, availability and the reliability for the replacement age around the optimal values. It is observed that the availability value is very close whether the replacement age is at 900 hours or at 1600 hours. This is mainly attributable to the small values of the downtimes for the preventive replacement,  $R_p$ , and failure replacement,  $R_f$ .

The three measures of effectiveness as discussed above are important criteria to determine the amount of spares required to be carried onboard or prepositioned at forward operating areas. When a ship is assigned to an operational area, say for 2 months, away from the homeport, the ship will have to rely on the forward base for replenishment. The replenishment cycle occurs every two or three weeks and usually takes about two days. So if the optimal replacement age is close (not necessarily within) to these stand-off periods we can undertake the replacement actions during these periods. It should be noted that if we do not carry the replacement action when it is due then if we continue to delay these actions everytime they are due, on the long run we are going to experience high costs, poor availability and low reliability.

Since we have three measures of effectiveness and the respective optimal replacement ages, it is up to the Decision Maker to decide which one he is going to give top priority.

**TABLE 3.1 VALUES OF COST/HR, AVAILABILITY AND  
RELIABILITY FOR VALUES OF REPLACEMENT AGE CLOSE TO  
THE OPTIMUM.**

<b>Replacement Age (<math>t_p</math>)</b>	<b>Cost (\$/hr)</b>	<b>Availability (A)</b>	<b>Reliability (R)</b>
900.0	32.78	0.9884	0.9779
950.0	31.90	0.9886	0.9755
1000.0	31.18	0.9887	0.9729
1050.0	30.59	0.9888	0.9702
1100.0	30.12	0.9888	0.9674
1150.0	29.75	0.9888	0.9644
1200.0	29.47	0.9888	0.9614
1250.0	29.26	0.9887	0.9582
1300.0	29.12	0.9886	0.9549
1350.0	29.02	0.9886	0.9515
1400.0	28.97	0.9885	0.9480
1450.0	28.95	0.9884	0.9443
1500.0	28.96	0.9883	0.9405
1550.0	28.99	0.9882	0.9367
1600.0	29.04	0.9881	0.9327

## B. SENSITIVITY ANALYSIS

Since we do not actually know the failure parameters, it is also important to do sensitivity analysis and see how the various measures of effectiveness vary with changes in those parameters. This will enable us to determine the parameters that we need to estimate very accurately.

We have assumed that the failure distribution is Weibull. Therefore, the parameters of interest are the shape parameter  $\alpha$  and the scale parameter  $\lambda$ . Besides these, other parameters are the expected costs and the downtime of preventive and failure replacement. In order to see the changes in the measures of effectiveness we hold all the other variables constant and only vary the parameter of interest. It would be possible to use Experimental Design techniques such as the Factorial Designs to study the main effects on the respective measures of effectiveness. This may help us to identify the parameters that are more sensitive and shall be estimated accurately.

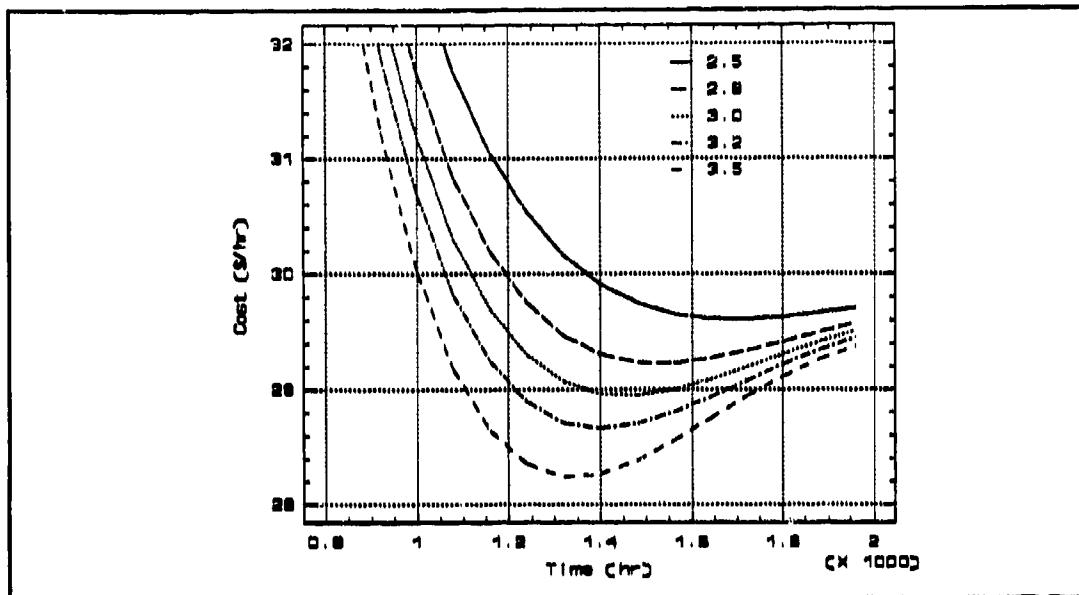
### 1. Alpha

Table 3.2 shows the optimal value of  $t_p^*$  for the minimum cost rate, the maximum availability and the reliability of 0.95 for sustaining a mission of duration 24 hours at age  $t$ ; and the respective optimal measure of effectiveness for various values of the shape parameter alpha. The graphical plots are shown in Figures 3.6, 3.7 and 3.8 for the cost rate, availability and the reliability functions respectively. It is observed that as the value of  $\alpha$  is

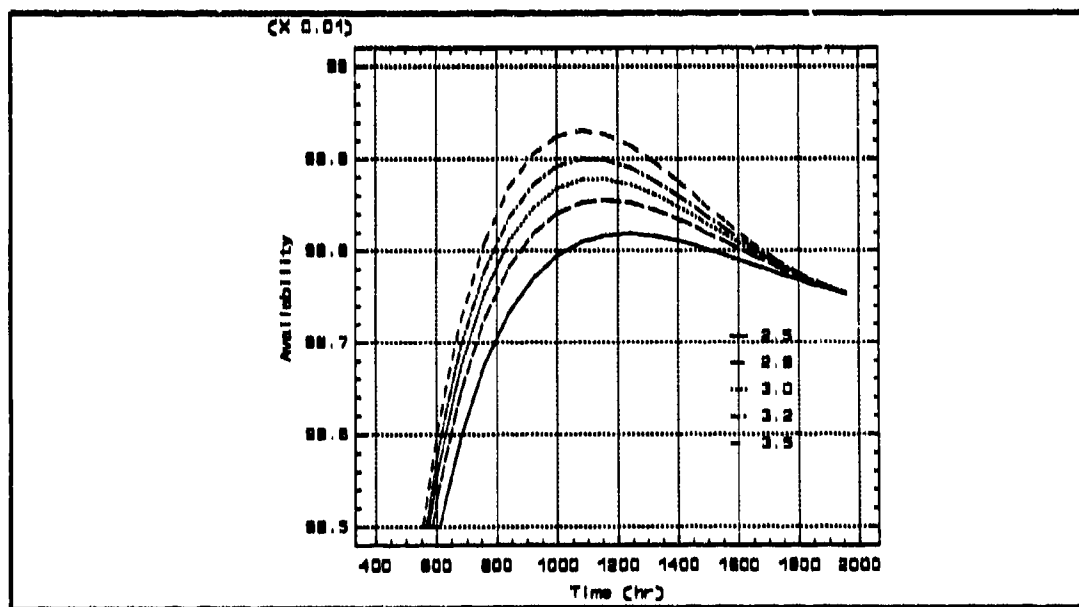
**TABLE 3.2 OPTIMAL REPLACEMENT AGE FOR VARIOUS  
VALUES OF ALPHA**

<b>ALPHA</b>	<b>COST</b>		<b>AVAILABILITY</b>		<b>RELIABILITY</b>	
	$t_p^*$	\$/hr	$t_p^*$	A	$t_p^*$	R
2.5	1691.8	29.62	1228.2	0.9882	1547.4	0.95
2.8	1526.0	29.23	1156.6	0.9886	1424.5	0.95
3.0	1453.5	28.95	1126.4	0.9888	1371.1	0.95
3.2	1399.3	28.67	1104.9	0.9890	1331.8	0.95
3.5	1340.7	28.24	1083.4	0.9893	1289.7	0.95

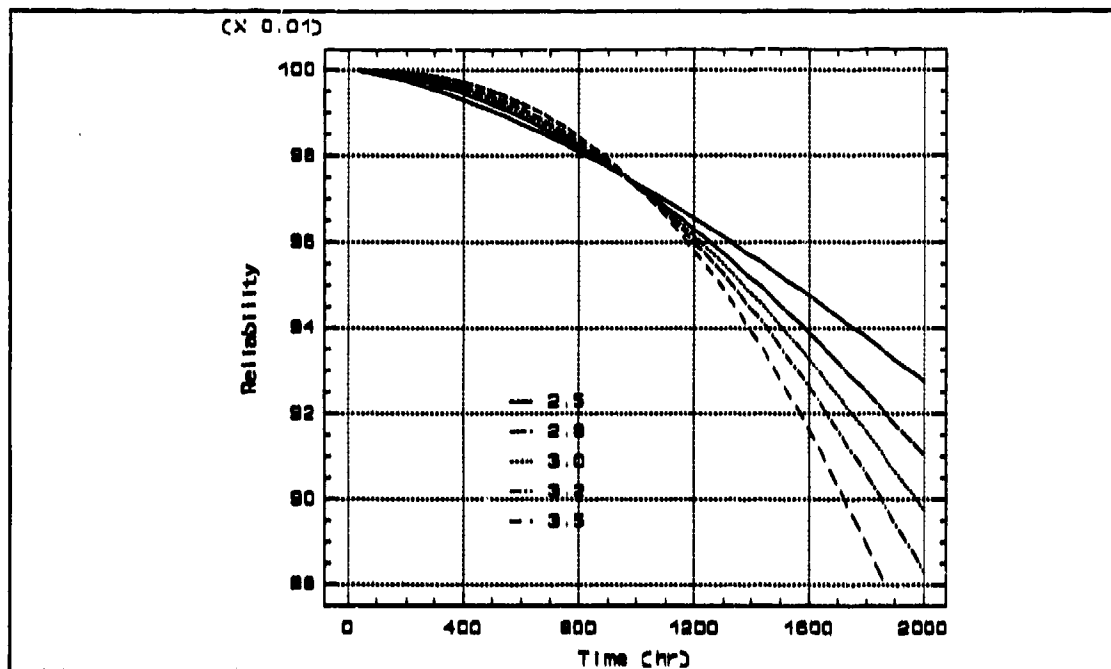
increased the replacement age gets shorter in order to minimize the costs and maximize the availability. This is also obvious from Figure 3.2 where the hazard increases as alpha increases and as such the failure rate also increases, which in turns requires replacement action early so as to optimize the respective measures of effectiveness. From Figure 3.7 it is observed that the curves are much flatter for low values of  $\alpha$  and for  $\alpha = 3.5$  the curve falls quite rapidly on both sides of the optimal value. Figure 3.8 shows that if we want a minimum reliability of 0.974 then the optimal replacement age is approximately 980 hours and at this value of reliability it is insensitive to the shape parameter alpha.



**Figure 8.6 Plot of the Cost Rate Function for Values of  $\alpha$ .**



**Figure 8.7 Plot of the Availability Function for Values of  $\alpha$ .**



**Figure 3.8 Plot of the Reliability Function for Values of Alpha.**

From Table 3.2, it is observed that if the true value of the shape parameter  $\alpha$  was 3.2 and in the analysis we estimated it to be 3.0, then we would have lost an availability of 0.0002 and would have incurred additional cost of \$0.28 per hour. On the other, if we have estimated it to be 3.5 then we would have gained an availability of 0.0003 at a lower cost by \$0.43 per hour. Therefore it is much better to estimate the shape parameter higher than lower and hence carry out the replacement or overhaul action earlier than later.

## 2. Lambda

Table 3.3 shows the optimal value of  $t_p^*$  and the respective optimal values for the cost, availability and 0.95 reliability to sustain a mission of 24

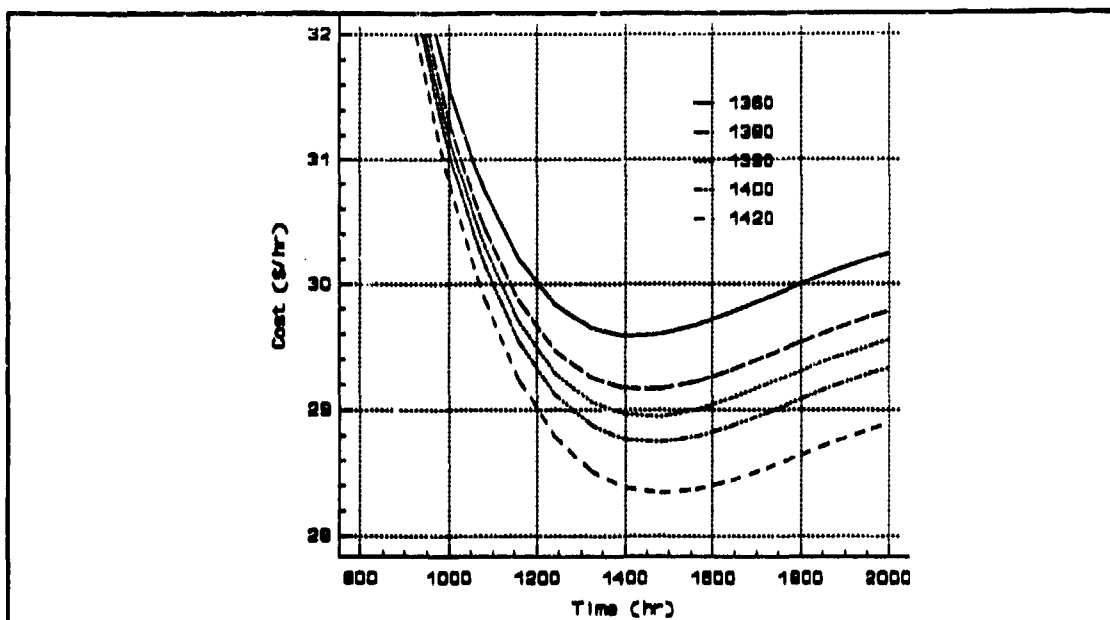


hours duration. The graphical plots are shown in Figures 3.9, 3.10 and 3.11 for the cost rate, availability and the reliability functions respectively.

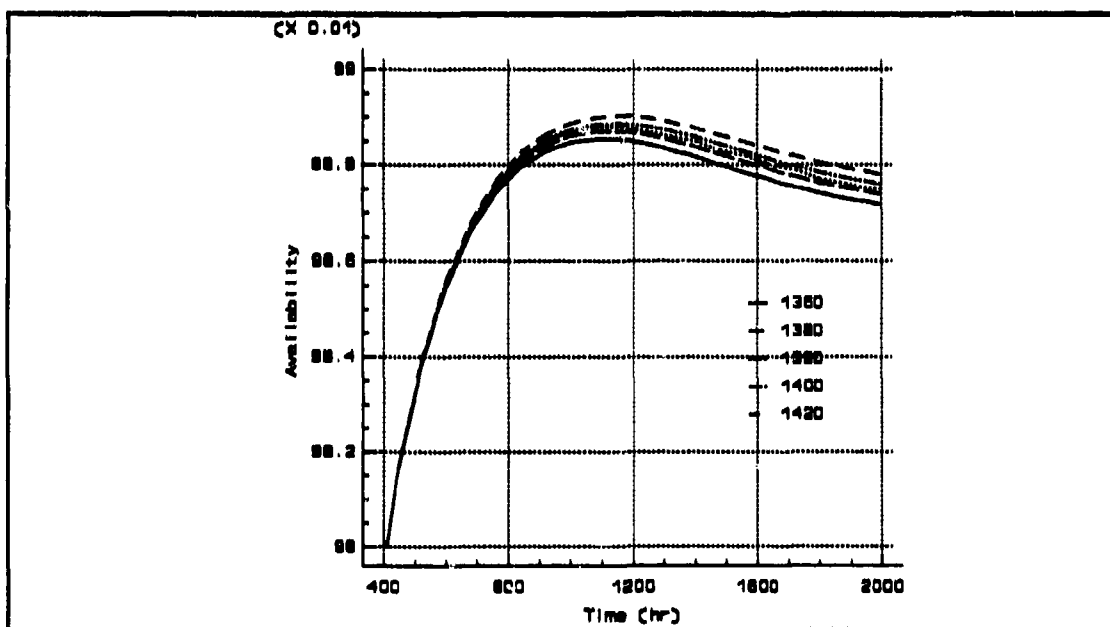
**TABLE 3.3 OPTIMAL REPLACEMENT AGE FOR VARIOUS VALUES OF LAMBDA (NOTE:  $\mu = 1/\text{LAMBDA}$ )**

MU	COST		AVAILABILITY		RELIABILITY	
	$t_p^*$	\$/hr	$t_p^*$	A	$t_p^*$	R
1360	1422.2	29.58	1102.1	0.9885	1326.7	0.95
1380	1443.0	29.16	1118.3	0.9887	1356.3	0.95
1390	1453.5	28.95	1126.4	0.9888	1371.1	0.95
1400	1463.9	28.75	1134.5	0.9889	1386.1	0.95
1420	1484.7	28.35	1150.7	0.9890	1416.2	0.95

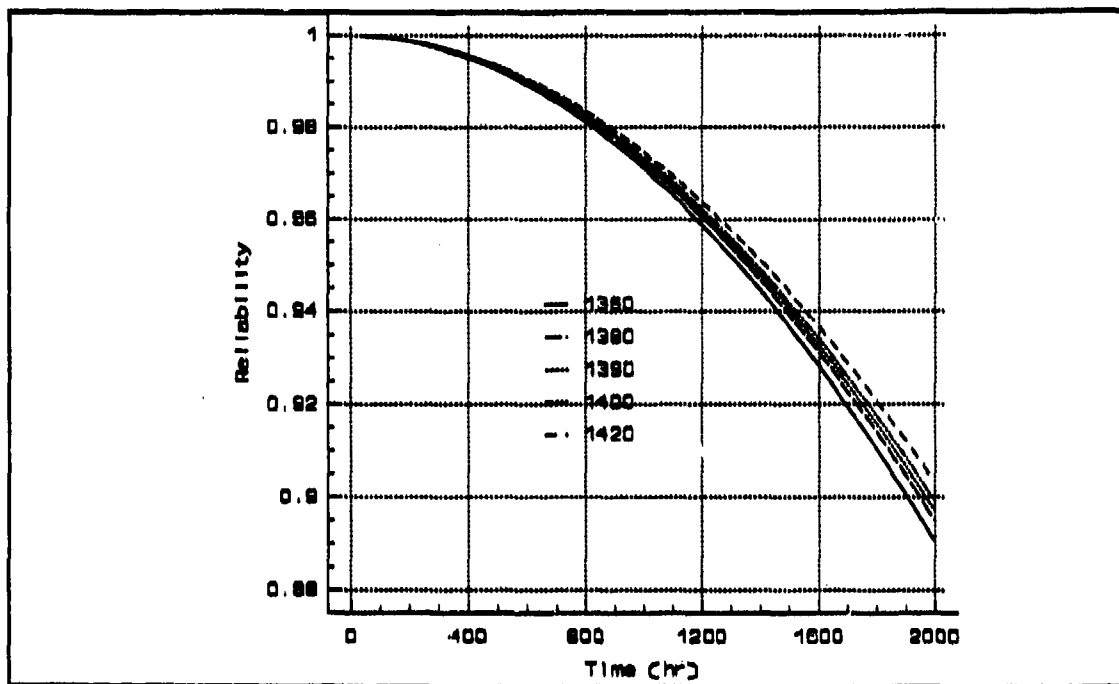
From the above table it is observed that the optimal replacement age for achieving a reliability of 0.95 when subjected to a mission of duration of 24 hours increases as  $\mu$  increases. This is because the higher the value of  $\mu$ , the reliability becomes better. Also when the value of  $\mu$  increases the optimal replacement age becomes longer for both the cost rate and the availability. The higher the value of  $\mu$  the failure rate for the Weibull distribution decreases and therefore the optimal replacement age becomes longer. A small difference in the value of  $\mu$  do not affect the replacement age drastically. However this is not



**Figure 3.9 Plot of the Cost Rate Function for Values of  $\mu$ .**



**Figure 3.10 Plot of the Availability Function for Values of  $\mu$ .**



**Figure 8.11 Plot of the Reliability Function for Values of  $\mu$ .**

true for the case of  $\alpha$ . Therefore, the shape parameter  $\alpha$  need to be estimated more accurately than the scale parameter  $\lambda$ .

### **3. Costs**

The costs for the failure replacement and the preventive replacement only affects the cost rate function. These costs estimates are quite easily available and they are usually fairly accurate based on previous data. Since we have not discounted costs and cash flow problems in the future it is necessary to observe the effects of the costs on the cost rate function. It is assumed that the increase in costs are proportional such that the ratio of replacement costs,  $C_f/C_p$ , is the same. It is observed that the replacement age does not vary as

long as the ratio of the replacement costs,  $C_f/C_p$ , remains the same. Equation (3.7) clearly shows that if the ratio of  $C_f/C_p$  is the same, there is no effect on the optimal replacement age  $t_p^*$ . However the cost per unit time increases proportionally as the replacement costs increases.

#### 4. Downtime

The availability function greatly depends on the expected downtime of the failure replacement ( $R_f$ ) and preventive replacement ( $R_p$ ) assuming the parameters for the Weibull distribution are estimated accurately. In our example we have taken  $R_f = 16$  hours and the  $R_p = 8$  hours. Because these figures are small compared to the optimal replacement age the availability is very high. In reality a failure at sea may take days to repair taking all the logistics and administrative requirements into considerations, especially when the ship is not accompanied by any auxiliary vessel or support ship. That is the reason all vital equipments onboard have redundancies incorporated. We can carry spares onboard but may not have other support elements to rectify the defect. Usually the downtime for a preventive replacement is constant because it is done at the base with all the support elements and the work is repetitive following standard procedures. Therefore, in our analysis we shall only vary  $R_f$  to see its effect on the availability. Table 3.4 shows the values of the availability if we carry out the preventive replacement at 1126 hours. It is observed that the availability of the system drops as  $R_f$  increases, but is still quite high.

**TABLE 3.4 AVAILABILITY FOR VARIOUS VALUES OF  
 $R_T$  AT  $T_P = 1126$ .**

<b>Expected Downtime <math>R_T</math> (hours)</b>	<b>Availability A</b>
<b>24</b>	<b>0.985</b>
<b>48</b>	<b>0.975</b>
<b>72</b>	<b>0.965</b>
<b>96</b>	<b>0.956</b>

### **C. EFFECTS OF SIMPLE PREVENTIVE MAINTENANCE**

In the above policy it was assumed that simple preventive maintenance actions did not improve the operational reliability of the system. In real life it is evident that these preventive maintenance actions may definitely enhance the system condition and, hence the reliability, but it will not restore the system to the original state i.e. to be as good as new. This was also emphasised in [Ref. 14]. However in reality there are situations for which preventive maintenance actions can degrade the system, that is by imperfect repair. Here we will concentrate on those actions that will improve the condition by a certain factor. So now we shall incorporate these preventive maintenance actions into our model. The questions to be asked is that when should we do these type of maintenance? How do we determine the improvement factor?

We can schedule the simple preventive maintenance actions whenever the operational reliability hits a minimum acceptable limit  $R_{\min}$  say 0.75 and then as mentioned above our reliability will improve by a certain factor, thus making the system 'younger' but not necessarily new. Or we could also carry out a planned replacement when the operational reliability hits the  $R_{\min}$  value and bring the condition of the system back to new. The appropriate maintenance action to be taken at these critical times depends on the measure of effectiveness required.

[Ref. 15] solved this problem by minimizing the expected cost rate as the measure of effectiveness. The number of simple preventive maintenance actions before a planned replacement action which gives a minimum cost rate is then obtained by evaluating the cost rate whenever the system reaches  $R_{\min}$ . Here we shall carry out the appropriate maintenance actions so as to maximize the availability of the system.

Notation is as follows:

1P	simple preventive maintenance
2P	planned preventive replacement
2C	failure replacement
T	time to failure
$R_i(t_i)$	probability of no failure during $(t_{i-1}, t_i)$
$F_i(t_i)$	probability of a failure during $(t_{i-1}, t_i)$
$F(t)$	probability distribution function of the time to failure

$f(t)$	probability density function of the time to failure
$h(t)$	failure rate function of the time to failure
$R_{ij}$	time required for each maintenance action $i=1,2$ $j=P,C$
$C_{ij}$	cost of each maintenance action $i=1,2$ $j=P,C$
$R_{min}$	minimum acceptable operational reliability limit
$t_i$	$i_{th}$ time the system reaches $R_{min}$

Let  $I$  be the improvement factor, then if maintenance type 1P is carried out at time  $t_1$  then the system age is reduced from  $t_1$  to  $(t_1 \cdot I)$  and therefore

$$t_2 = t_1 + t_1 (1 - I)$$

and 
$$t_3 = t_2 + (t_2 - t_1) (1 - I)$$

in general 
$$t_n = t_{n-1} + (1 - I)^{n-1} t_1 \quad (3.18)$$

As  $I \rightarrow 1$ , the effect on the system approaches "bad as old" and

As  $I \rightarrow 0$ , the effect approaches "good as new" [Ref. 15].

The value of  $t_1$  can be computed as follows:

$$t_1 = \frac{\{-\ln R_{min}\}^{\frac{1}{\lambda}}}{\lambda} \quad (3.19)$$

and from equation (3.19) we can compute  $t_i$  for  $i = 2, 3, \dots, n$ .

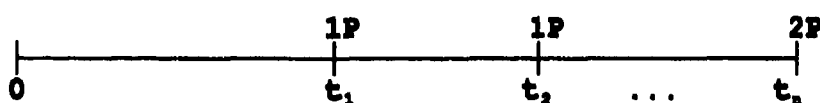
The value of  $I$  can be estimated from past records from the data collected on similar equipment where performance measurements are taken before and after the simple preventive maintenance actions. In some cases performance measurement techniques such as condition monitoring by vibration analysis are employed. This is largely used in rotating machinery. As mentioned earlier,

simple preventive maintenance actions constitutes cleaning, adjustments, replacing worn out parts, tuning and minor calibration. All these actions normally improve the reliability which is a function of the failure rate. So these actions may prolong the life of the system, and enable the system to be 'younger'. From experience, it is found that as the system becomes older these simple preventive maintenance actions becomes ineffective and as such the system may require a complete overhaul which on completion is as good as new. For our case here we shall consider that on completion of each simple preventive maintenance action, the system age is reduced from  $t_i$  to  $t_i - (t_i - t_{i,1}) (1 - I)$  where  $t_i$  is the time of the simple preventive maintenance action.

A cycle is completed each time a replacement takes place which could be either due to a failure replacement or a planned replacement. On completion of the replacement action the cycle probabilistically starts all over.

### 1. Case 1

$T > t_n$  (Failure occurs after the type 2P maintenance)



The costs associated are:

a) Expected costs of type 1P maintenance.

If planned replacement takes place at  $t_n$ , then there will be  $(n - 1)$  maintenance actions of type 1P and each costs  $C_{1P}$ .



Then total cost =  $(n - 1) C_{1P}$

But this event will occur if there are no failures in the first  $(n-1)$  intervals. The Probability of this event is =  $R_1(t_1)R_2(t_2)...R_n(t_n)$ . It is assumed that the failures are statistically independent. The failure in each interval is independent from the failure in next interval.

Therefore expected costs of type 1P maintenance =

$$(n - 1) \cdot C_{1P} \cdot \prod_{i=1}^n R_i(t_i) \quad (3.20)$$

where  $R_i(t_i) = 1 - \text{Probability of a failure in the } (n-1) \text{ intervals and is given by}$

$$R_i(t_i) = 1 - [F(t_i) - F(t_i - (t_i - t_{i-1}))] \quad (3.21)$$

b) Expected costs of type 2P maintenance.

Since there will be only one Type 2P maintenance in an interval, the expected costs of type 2P maintenance =

$$C_{2P} \cdot \prod_{i=1}^n R_i(t_i) \quad (3.22)$$

The expected downtime can be obtained by substituting the cost by downtime in equations (3.19) and (3.20).

The downtime associated are:

a) Expected downtime of type 1P maintenance =

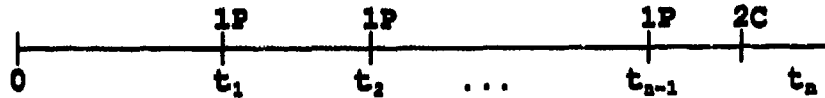
$$(n - 1) \cdot R_{1P} \cdot \prod_{i=1}^n R_i(t_i) \quad (3.23)$$

b) Expected downtime of type 2P maintenance =

$$R_{2P} \cdot \prod_{i=1}^n R_i(t_i) \quad (3.24)$$

## 2. Case 2

$$t_{n-1} < T < t_n$$



The costs associated are:

a. Expected cost of type 1P maintenance

If the failure occurs between  $t_{n-1}$  and  $t_n$ , then there will be  $(n-1)$  preventive maintenance actions and the Probability of this event is = Probability of no failures in the first  $(n-1)$  intervals multiplied by the Probability of a failure between  $t_{n-1}$  and  $t_n$ . Again we assume that failures are independent. Then the expected cost of type 1P maintenance =

$$\sum_{i=1}^n [(i-1) \cdot C_{1P} \cdot \prod_{j=1}^{i-1} R_j(t_j) F_i(t_i)] \quad (3.25)$$

b. Expected costs of type 2C maintenance =

$$\sum_{i=1}^n [C_{2C} \cdot \prod_{j=1}^{i-1} R_j(t_j) \cdot F_i(t_i)] \quad (3.26)$$

The downtime associated are:

a. Expected downtime of type 1P maintenance =

$$\sum_{i=1}^n [(i-1) \cdot R_{1p} \cdot \prod_{j=1}^{i-1} R_j(t_j) \cdot F_i(t_i)] \quad (3.27)$$

b. Expected downtime of type 2C maintenance =

$$\sum_{i=1}^n [R_{2c} \cdot \prod_{j=1}^{i-1} R_j(t_j) \cdot F_i(t_i)] \quad (3.28)$$

The expected operation time until  $t_n$  is the sum of the average operation time until  $t_{n-1}$  and the average operation time during  $(t_{n-1}, t_n)$ .

The expected downtime is the sum of all the downtimes in equations (3.23), (3.24), (3.27) and (3.28).

From our earlier discussion we know that the mean life of the system, assuming only 1 interval, is given by

$$M(t_1) = \int_0^{t_1} [1 - F(t)] dt \quad (3.29)$$

and the above equation can also be written as

$$M(t_1) = \int_0^{t_1} t f(t) dt + R(t_1)t_1 \quad (3.30)$$

and for  $n$  intervals the equation can be written as

$$\sum_{i=1}^n \left\{ \prod_{j=1}^{i-1} R_j(t_j) \cdot \left[ \int_{t_{i-1}}^{t_i} t f(t) dt + R_i(t_i)(t_i - t_{i-1}) \right] \right\} \quad (3.31)$$

$$\text{Now Availability} = \frac{\text{Total Operation Time}}{\text{Total Operation Time} + \text{Total Downtime}}$$

It will be shown later that if the expected downtime for a type 1P maintenance (simple preventive maintenance) is small then the availability function given by the above equation is a good approximation.

The expected costs is the sum of all the costs in equations (3.20), (3.22), (3.25) and (3.26). Then the cost rate function,  $C(t_p)$ , is given by

$$C(t_p) = \frac{\text{Total Expected Costs}}{\text{Total Operation Time} + \text{Total Downtime}}$$

### 3. Numerical Illustration

Now let us take a case example with the following data:

Let the time to failure,  $T$ , follow a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$ .

Let

$$\alpha = 3.0$$

$$\lambda = 1/1390$$

$$R_{1P} = 1 \text{ hour}$$

$$R_{2P} = 8 \text{ hours}$$

$$R_{3C} = 48 \text{ hours}$$

Using the above data and equations, a Fortran program was written [Annex A] to determine the optimum times to carry out simple preventive maintenance actions and complete overhauls or preventive replacement so as

to maximize the availability. The results are tabulated in Tables 3.5, 3.6, and 3.7 for values of  $I = 0.1, 0.5$  and  $0.9$  respectively.

When  $I = 0.1$ , carry out simple preventive maintenance actions at times:

- 917.6 hours
- 1743.4 hours
- 2486.7 hours

and preventive replacement or complete overhaul at time 3155.6 hours to achieve a maximum availability of 0.9836.

When  $I = 0.5$ , carry out simple preventive maintenance action at time 917.6 hours and a preventive replacement at time 1376.4 hours to achieve a maximum availability of 0.9802.

When  $I = 0.9$ , do not carry out any simple preventive maintenance action and at time 917.6 hours replace or carry a complete overhaul of the system so as to achieve a maximum availability of 0.9794.

It is observed that as the improvement factor,  $I \rightarrow 0$ , the system becomes "good as new" after each simple preventive maintenance action and as the improvement factor,  $I \rightarrow 1$ , the system becomes "bad as old" after each preventive maintenance action.

**TABLE 3.5 REPLACEMENT TIMES FOR IMPROVEMENT FACTOR,  
 $I = 0.1$  AND  $R_{1P} = 8$  HOURS,  $R_{2C} = 48$  HOURS,  $R_{1P} = 1$  HOUR,  $\alpha = 3.0$ ,  
 $\mu = 1390$  HOURS.**

<b>N</b>	<b>TIME (HOURS)</b>	<b>AVAILABILITY</b>
1	917.6	0.979421
2	1743.4	0.982197
3	2486.7	0.983305
4	3155.6	0.983591
5	3757.7	0.983585
6	4299.5	0.983480
7	4787.1	0.983344
8	5226.0	0.983206
9	5621.0	0.983075
10	5976.5	0.982955
11	6296.5	0.982848
12	6584.4	0.982751
13	6843.6	0.982664
14	7076.8	0.982587
15	7286.7	0.982518

**TABLE 3.6 REPLACEMENT TIMES FOR IMPROVEMENT FACTOR,  
I = 0.5 AND  $R_{1P}$  = 8 HOURS,  $R_{2C}$  = 48 HOURS,  $R_{1P}$  = 1 HOUR,  $\alpha$  = 3.0,  
 $\mu$  = 1390 HOURS.**

<b>N</b>	<b>TIME (HOURS)</b>	<b>AVAILABILITY</b>
1	917.6	0.979421
2	1376.4	0.980155
3	1605.8	0.980074
4	1720.5	0.979848
5	1777.8	0.979644
6	1806.5	0.979483
7	1820.9	0.979358
8	1828.0	0.979261
9	1831.6	0.979184
10	1833.4	0.979122
11	1834.3	0.979071
12	1834.7	0.979029
13	1835.0	0.978994
14	1835.1	0.978964
15	1835.1	0.978938

**TABLE 3.7 REPLACEMENT TIMES FOR IMPROVEMENT FACTOR,  
 $I = 0.3$  AND  $R_{xp} = 8$  HOURS,  $R_{xc} = 48$  HOURS,  $R_{ip} = 1$  HOUR,  $\alpha = 3.0$ ,  
 $\mu = 1390$  HOURS.**

<b>N</b>	<b>TIME (HOURS)</b>	<b>AVAILABILITY</b>
1	917.6	0.979421
2	1009.4	0.979042
3	1018.5	0.978456
4	1019.5	0.977955
5	1019.5	0.977540
6	1019.6	0.977193
7	1019.6	0.976899
8	1019.6	0.976646
9	1019.6	0.976426
10	1019.6	0.976123



#### IV. POLICY II (MINIMAL REPAIR WITH AGE REPLACEMENT)

As an alternative assume that the system consists of several components and the system fails when one of the components fail, that is components are connected in series. We shall assume the components have independent IFR distribution. Then the system lifetime is also IFR. (The IFR property is maintained under the formation of the series system). Preventive replacement is carried out when the system reaches the age  $t_p$ . Between the preventive maintenance, failures are repaired as quickly as possible (minimal repairs) either by replacing the failed component with a new one or repairing the failed part. We also assume that the system failure rate is not disturbed on completion of the minimal repair i.e. the failure rate is the same as before failure. If the failure occurs at time  $t < t_p$ , the failure rate of the system just after the minimal repair is  $h(t)$ . Failures are detected immediately.

This idea of minimal repair was first introduced by Barlow and it is described in [Ref. 7]. After that, many authors expanded on these ideas and formulated various models associated with minimal repairs; they used the expected cost rate as the measure of effectiveness. In this policy we will emphasize the availability of the system as our measure of effectiveness. However we will also formulate the expected cost rate models.

Let  $0 \leq t$  be the system operating time since last replacement. Then the probability of a failure occurring in  $[t, t+dt]$  is  $h(t)dt$  where  $h(t)$  is the failure rate of the system.

Let  $N(t_p)$  = number of failures occurring in time  $(0, t_p)$

When repairs are minimal,  $\{ N(t_p), t_p \geq 0 \}$  is a Nonhomogeneous Poisson Process with intensity function  $h(t_p)$ .

The probability that the system will experience  $n$  failures in the interval  $(0, t_p)$  is given by

$$P\{ N(t_p) = n \} = \frac{[m(t_p)]^n e^{-m(t_p)}}{n!} \quad (4.1)$$

where

$$m(t_p) = E[ N(t_p) ] = \int_0^{t_p} h(t) dt \quad (4.2)$$

$m(t_p)$  is called the *mean value function*.

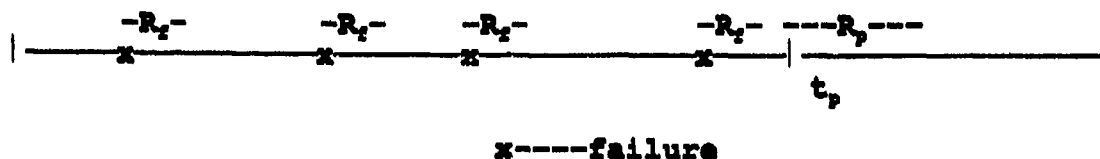
## A. COST MODEL

The age  $t_p^*$  at which the operating system is replaced depends on the following factors:

- Failure distribution (IFR).
- Costs of minimal repair and preventive replacement.
- Downtime of minimal repair and preventive replacement.

- Measure of effectiveness:
  - Minimize costs.
  - Maximize availability.

We have assumed that there are no simple preventive maintenance actions.



Let  $C_r$  be the mean cost of each minimal repair.

$C_p$  be the mean cost of preventive replacement.

$R_p$  be the mean time of preventive replacement.

$R_r$  be the mean time of a minimal repair.

and  $t_p^*$  be the planned replacement age.

$$C_r > C_p \quad \text{and} \quad R_r > R_p$$

Then, assuming that the cost of downtime is negligible, the total cost is given by

$$C = \sum_{i=1}^{N(t_p)} C_{ri} + C_p \quad (4.3)$$

where  $C_{ri}$  is the cost of the  $i$ -th minimal repair and the length of the cycle is given by

$$L = t_p + R_p \quad (4.4)$$

By using equation (3.1) and assuming that the time for a minimal repair is very small compared to that of the length of a cycle, the expected cost rate function is given by

$$C(t_p) = \frac{C_f \int_0^{t_p} h(t) dt + C_p}{t_p + R_p} \quad (4.5)$$

and we wish to minimize the cost per unit time, so we set the derivative of equation (4.5) to 0 and we obtain

$$h(t_p)[t_p + R_p] - \int_0^{t_p} h(t) dt = \frac{C_p}{C_f} \quad (4.6)$$

Then the value of  $t_p^*$  that satisfies the above equation is the optimal replacement age.

For a time to failure,  $T$ , following a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$  the expected cost rate function is given as below

$$C(t_p) = \frac{C_f (\lambda t_p)^\alpha + C_p}{t_p + R_p} \quad (4.7)$$

For given values of  $C_f$ ,  $C_p$ ,  $R_p$ ,  $R_f$  and the parameters  $\alpha$  and  $\lambda$  we can find the optimal values of  $t_p^*$  that minimize the expected cost rate.

Now let us take a case example with the data as follows, but first without preventive maintenance action:

$$C_p = \$ 25000$$

$$C_r = \$ 1000$$

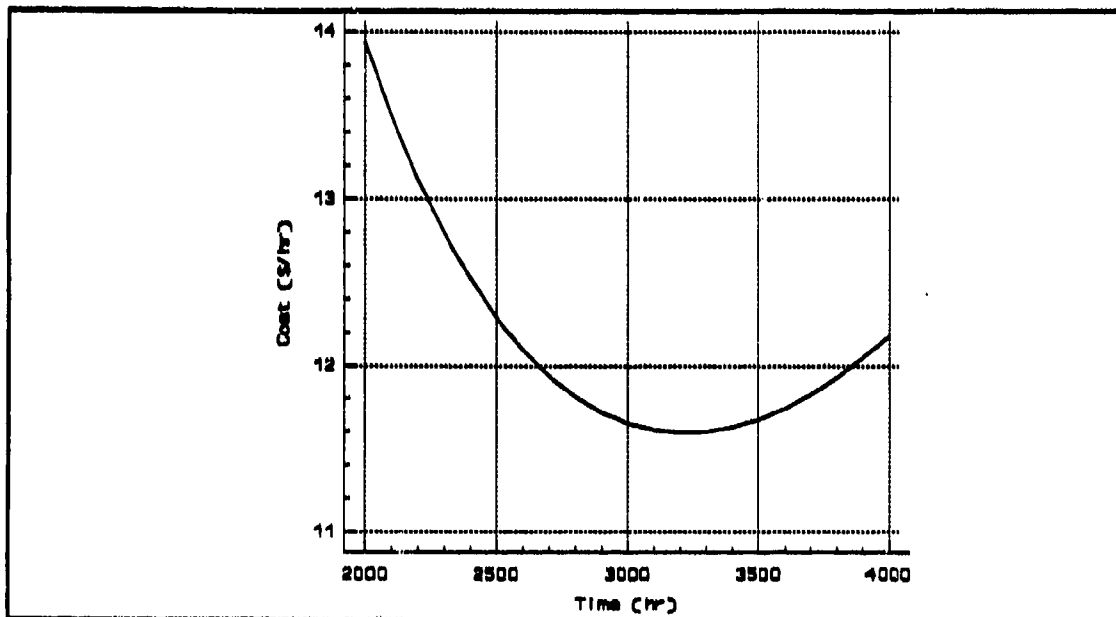
$$R_p = 8 \text{ hours}$$

$$R_r = 1 \text{ hour}$$

$$\alpha = 3.0$$

$$\lambda = 1/1390 \text{ hours}$$

Figure 4.1 is a plot of the cost rate function as in equation (4.7) and the optimal value of  $t_p^*$  that minimizes the cost rate function is 3222 hours at a cost of \$11.60/hour.



**Figure 4.1 Plot of the Cost Rate Function.**

Table 4.1 shows the values of the cost rate for the replacement age  $t_p$  close to the optimal value. From Figure 4.1 it is observed that the curve is fairly flat near the optimal value thus giving some flexibility to the decision maker.

**TABLE 4.1 VALUES OF COST/HOUR FOR VARIOUS REPLACEMENT AGE.**

<b>Replacement Age (hours)</b>	<b>Cost (\$/hour)</b>
3000.0	11.65
3050.0	11.63
3100.0	11.61
3150.0	11.60
3200.0	11.60
3250.0	11.60
3300.0	11.60
3350.0	11.61
3400.0	11.63
3450.0	11.65
3500.0	11.68

Now suppose there are  $s$  identical components which operate and fail independently and have the same failure distributions with the same parameters. All of these components are replaced at time  $t_p$  at a cost of  $C_p$ . When each of the component fails it is replaced individually or undergoes

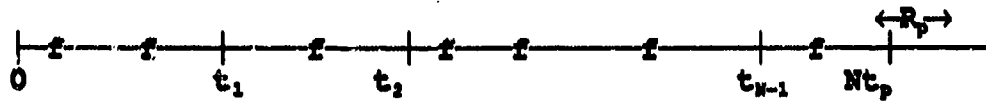
minimal repair without affecting the system failure rate as a whole. Then the expected cost rate function becomes

$$C(t_p) = \frac{s C_f \int_0^{t_p} h(t) dt + C_p}{t_p + R_p} \quad (4.8)$$

Now let us imagine that we carry out a simple preventive maintenance action at some time  $t_p$  and after  $N$  of these preventive maintenance actions are carried out we either replace the system or carry out a complete overhaul which on completion is like new. Any failures between the preventive replacement are treated as minimal failures and are repaired quickly in negligible downtime at a cost of  $C_f$ . The cost of each simple preventive maintenance action is  $C_p$ . It is assumed that after each simple preventive maintenance action the system improves by a certain factor, so for simplicity we say that the system becomes 'younger'. Nakagawa [Ref. 16]. This means that if  $t$  is the time of a preventive maintenance action the failure rate on completion is  $h(t-x)$  where  $x$  is the amount of time by which the system has become 'younger'. However the failure rate after a minimal repair stays the same. The value of  $x$  can perhaps be determined from past historical records using performance measurements techniques such as condition monitoring by vibration analysis or some output parameter. However a methodology for characterizing the effective age reduction,  $x$ , remains to be developed.

According to the above assumption the replacement age is  $Nt_p$ , and there are  $N-1$  simple preventive maintenance actions.

A cycle is completed at  $(Nt_p + R_p)$  and the process probabilistically starts all over.



- $f$  ---- minimal failure
- $R_p$  ---- expected time of preventive replacement
- $t_i, i = 1, 2, \dots, N-1$  ---- simple preventive maintenance
- $Nt_p$  ---- preventive replacement

( Note: At  $t_i$  the age of the system is  $(t_i - x)$  )

The total expected costs,  $E[C]$ , incurred in a cycle is given by

$$E[C] = C_f \int_0^{t_1} h(t) dt + C_m + C_f \int_{t_1-x}^{t_1+(t_p-x)} h(t) dt + C_m + \dots + C_f \quad (4.9)$$

$$\text{for } 0 \leq x \leq t_p$$

and this equation can be simplified to

$$E[C] = C_f \sum_{i=0}^{N-1} \int_{(t_p-x)}^{t_p+(t_p-x)} h(t) dt + (N-1)C_m + C_p \quad (4.10)$$

$$\text{for } 0 \leq x \leq t_p$$

and then the expected cost rate function,  $C(t_p)$ , is given by

$$C(t_p) = \frac{E[C]}{E[L]}$$

where  $E[L] = Nt_p + R_p$ .



For a time to failure,  $T$ , following a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$  the expected cost rate function is given as follows

$$C(t_p) = \frac{C_f \sum_{i=0}^{N-1} \int_{(t_p-x)^i}^{t_p+(t_p-x)^i} \alpha \lambda^{\alpha} t^{\alpha-1} dt + (N-1)C_{mp} + C_p}{Nt_p + R_p} \quad (4.11)$$

When  $C_{mp} = 0$  and  $x = 0$ , that is there is no preventive maintenance action, then the above equation is the same as equation (4.7).

Now we shall plan to carry out a preventive maintenance at some time  $t_p$  at a cost of  $C_{mp} = \$ 500.00$ . On completion of this simple preventive maintenance action, the age of the system becomes younger by some value  $x$ . When  $x = 0$ , it is observed that there is no improvement to the optimal planned replacement age but we have incurred an additional cost at \$11.75/hour instead of the original cost at \$11.60/hour. As  $x$  increases from 0, the optimal planned replacement age increases and also the cost rate reduces. This is for the case of carrying out only one preventive maintenance in between the planned replacement or overhaul age. We could carry on the same analysis incorporating more preventive maintenance actions and the results will give more improvement. However this greatly depends on the parameters such as the value of  $x$  and the cost of preventive maintenance assuming the other parameters remain fixed.

In the real scenario as the system gets older there is more deterioration and on completion of each preventive maintenance action the value of  $x$  probably reduces over time; that is; it is some function of  $t$ , and there comes a stage at which preventive maintenance action will not improve the system any longer. For simplicity, however we assume that  $x$  is constant. It should be noted that there will be values of age for which  $(\text{age} - x) < 0$ , we therefore take the maximum  $\{0, (\text{age} - x)\}$  as the age of the system on completion of a simple preventive maintenance action.

The Figure 4.2 and Table 4.2 illustrate that simple preventive maintenance actions increases the optimal replacement age.

Table 4.2 shows the time for a preventive replacement (TPR) and the time for a simple preventive maintenance (TPM) for various values of  $x$ .

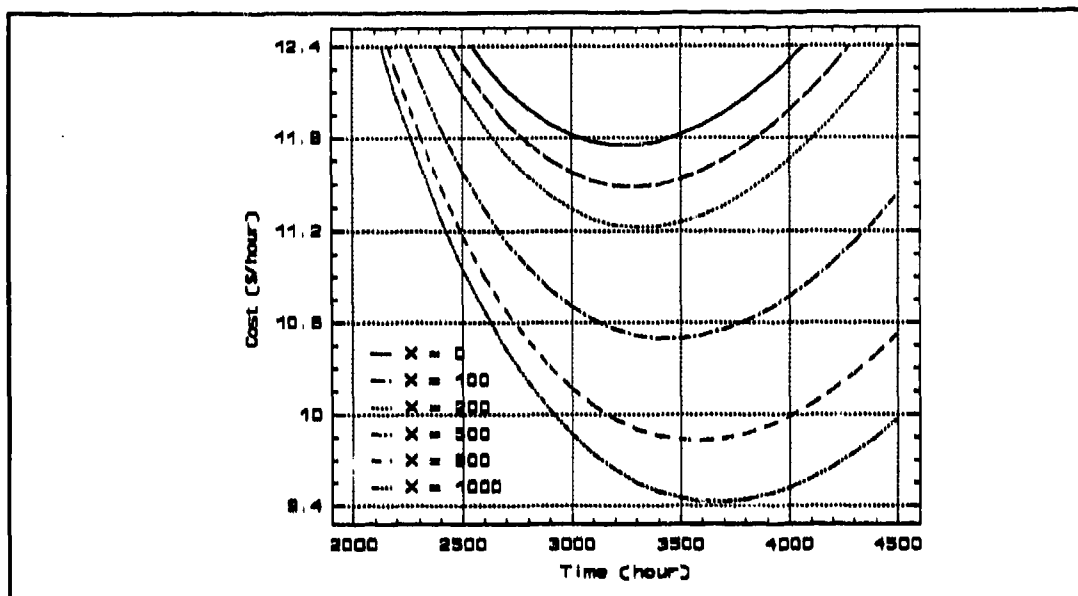
## B. AVAILABILITY MODEL

Now let us take the availability as our measure of effectiveness. From earlier discussion Availability is given by

$$A(t_p) = \frac{\text{Mean Life During A Cycle}}{\text{Total length of cycle}}$$

$$A(t_p) = \frac{t_p - R_p \int_0^{t_p} h(s) ds}{t_p + R_p} \quad (4.12)$$

and we wish to maximize the availability function, so we set the derivative of equation (4.12) to 0 and we obtain



**Figure 4.2 Plot of the Cost Function with Preventive Replacement for Various Values of  $x$ .  $C_{sp} = \$500.00$ .**

**TABLE 4.2 OPTIMAL PREVENTIVE REPLACEMENT AGE WITH PREVENTIVE MAINTENANCE FOR VARIOUS VALUES OF  $x$ .  $C_{sp} = \$500.00$ .**

$x$ (hours)	TPR (hours $\pm 1$ )	TPM (hours $\pm 0.5$ )	Cost (\$/hour)
0	3244	1622	11.75
100	3281	1641	11.48
200	3320	1660	11.22
500	3443	1722	10.49
800	3573	1787	9.83
1000	3667	1834	9.43

$$h(t_p) [t_p + R_p] - \int_0^{t_p} h(t) dt = \frac{R_p}{R_r} \quad (4.13)$$

Then the value of  $t_p^*$  that satisfies the above equation is the optimal replacement age.

For a time to failure,  $T$ , following a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$  the availability function is given as follows

$$A(t_p) = \frac{t_p - R_p(\lambda t_p)^\alpha}{t_p + R_p} \quad (4.14)$$

The availability function in equation (4.12) is an approximation because we do not take into account of the downtime during a minimal repair, which in reality is not entirely valid. In the equation we assume that the system can still fail when the system is down (we are integrating over  $(0, t_p)$  and some of that is downtime). However, this approximation is quite accurate if the downtimes are small, as is likely to be true in practice.

We can find an exact solution to equation (4.12) as follows:

Let

$A(t)$  = Probability the system is available at time  $t$  (age  $t$ ) following the last replacement.

$h(t)$  = failure rate at time  $t$ .

$\mu(t)$  = repair rate at time  $t$ . For our case this is assumed to be constant and denoted by  $\mu$ . Then  $\mu = R_r^{-1}$ .

(It should be noted that it may take longer to repair an older system)

Then,

$$A(t+dt) = A(t) \{1 - h(t)dt\} + \{1 - A(t)\}\mu(t)dt + o(dt) \quad (4.15)$$

where  $o(dt)$  represents higher order terms which are  $= 0$

From equation (4.15), arranging the terms we get

$$\frac{A(t+dt) - A(t)}{dt} = -h(t) A(t) + \mu(t) \{1 - A(t)\} \quad (4.16)$$

$$\frac{dA(t)}{dt} = -h(t) A(t) - \mu(t) A(t) + \mu(t) \quad (4.17)$$

$$\frac{dA(t)}{dt} + \{h(t) + \mu(t)\} A(t) = \mu(t) \quad (4.18)$$

$$\frac{d}{dt} \{A(t) e^{\int_0^t (h(t_1) + \mu(t_1)) dt_1}\} = \mu(t) e^{\int_0^t (h(t_1) + \mu(t_1)) dt_1} \quad (4.19)$$

$$\therefore A(t) e^{\int_0^t (h(t_1) + \mu(t_1)) dt_1} - A(0) = \int_0^t \mu(t_2) e^{\int_0^{t_2} (h(t_1) + \mu(t_1)) dt_1} dt_2 \quad (4.20)$$

$$A(t) = A(0) e^{-\int_0^t (h(t_1) + \mu(t_1)) dt_1} + \int_0^t \mu(t_2) e^{-\int_2^t (h(t_1) + \mu(t_1)) dt_1} dt_2 \quad (4.21)$$

For a time to failure,  $T$ , following a Weibull Distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$ , the failure rate  $h(t)$  is given by

$$h(t) = \alpha \lambda^\alpha t^{\alpha-1} \quad \alpha > 1, \lambda > 0, t \geq 0 \quad (4.22)$$

then substituting  $A(0) = 1$  and  $\mu(t) = (R_r)^{-1}$ ;  $A(t)$  can be simplified as follows:

$$A(t) = e^{-[(\lambda_0)^n + \frac{t}{R_p}]} + \frac{1}{R_p} \int_0^t e^{-[(\lambda_0)^n - (\lambda_0)^n + \frac{t}{R_p} - \frac{t_2}{R_p}]} dt_2 \quad (4.23)$$

But the integral  $\int_0^t e^{-[(\lambda_0)^n - (\lambda_0)^n + \frac{t}{R_p} - \frac{t_2}{R_p}]} dt_2$  has no closed form

solution. Therefore we have to evaluate it numerically.

Using the same principle as that in [Ref. 17] the average availability over a cycle of length  $t_p + R_p$  and hence in the long run is given by

$$A(t_p) = \frac{1}{t_p + R_p} \int_0^{t_p} A(t) dt \quad (4.24)$$

By taking the derivative of equation (4.19), we obtain

$$\frac{dA(t_p)}{dt_p} = \frac{(t_p + R_p) A(t_p) - \int_0^{t_p} A(t) dt}{(t_p + R_p)^2} \quad (4.25)$$

Setting

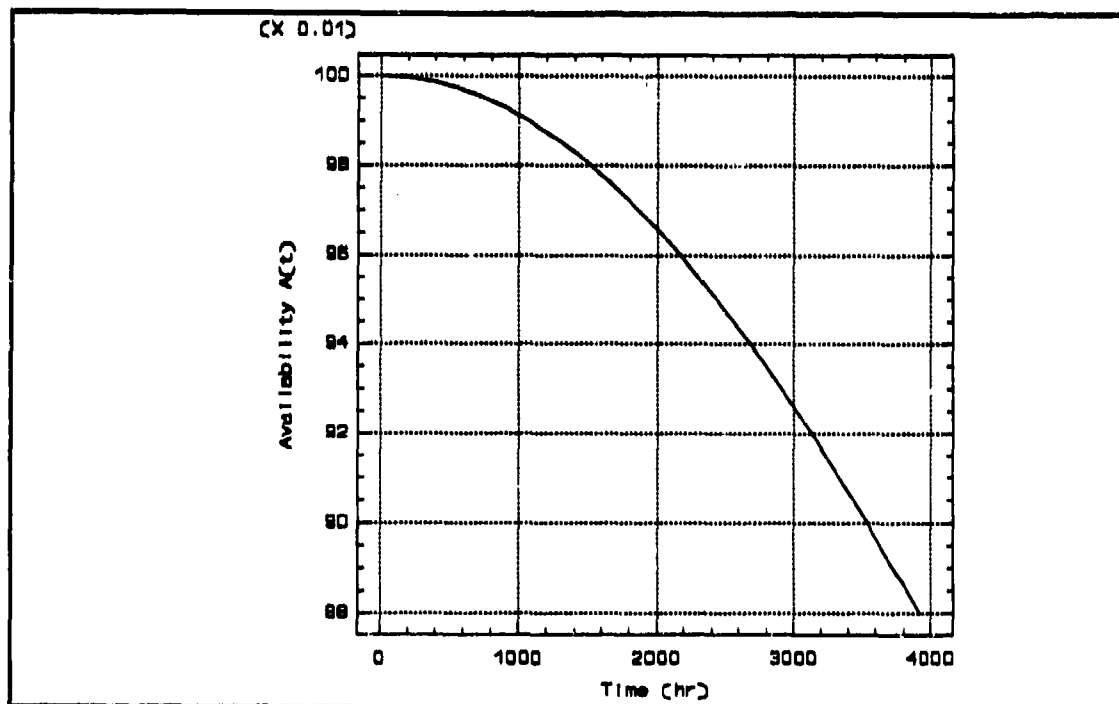
$$\frac{dA(t_p)}{dt_p} = 0, \text{ for } t_p^* > 0 \text{ then } t_p^* \text{ is a candidate for a time between the end of one preventive replacement and the beginning of the next}$$

and  $t_p^*$  is the value that satisfies the following equation

$$A(t_p^*) = \frac{1}{t_p^* + R_p} \int_0^{t_p^*} A(t) dt \quad (4.26)$$

To solve equation (4.24) the IMSL subroutine DQDAG was used to compute the integral in equation (4.23) for various values of  $t$ . So we obtained  $A(t)$  for various values of  $t$  and the graph of this function is shown in Figure

4.3. Then we used the IMSL subroutine DCSINT [Ref. 13] to compute the cubic spline interpolant to the set of data points obtained earlier (values of  $t$  and the corresponding values of  $A(t)$ ). Finally we used the IMSL subroutine DCSITG [Ref. 13] to evaluate the integral of the cubic spline for various values of  $t_p$ . A brief description of the IMSL subroutines DCSINT and DCSITG are in Appendix D and E respectively. The values obtained by using this 'exact' method was compared with those values obtained using the approximation as in equation (4.12). These values are tabulated below. It is observed that the



**Figure 4.3 Plot of the Availability Function for Values of  $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_f = 8$  hours,  $R_p = 8$  hours.**

both the approximate and exact solutions are the same to four decimal places and the approximation gives an accurate solution. This is true for small values of  $R_r$  and as  $R_r$  is increased from 1.0 hour to 8 hours, both the approximation and the exact solutions are the same to two decimal places, as shown in Table 4.4. However the optimal replacement age are the same for both the cases. As  $R_r$  increases the accuracy of the approximation diminishes.

Table 4.3 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_r = 17$  hours,  $R_p = 8$  hours. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	2203 hours	0.994581
• exact solution	2208 hours	0.994589

It is observed that the approximation gives very accurate results.

Table 4.4 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_r = 8$  hours,  $R_p = 8$  hours. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	1099 hours	0.989201
• exact solution	1108 hours	0.989301

It is again observed that the approximation gives very accurate results.



Table 4.5 shows the values of availability for  $\alpha = 2.5$ ,  $\mu = 1390$ ,  $R_r = 1$  hour,  $R_p = 8$  hours. The optimal replacement age for the two methods and the maximum availability are:

- approximate solution      2711 hours      0.995104
- exact solution              2717 hours      0.995112

Once again it is observed that the approximation gives very accurate results. If the actual shape parameter was 2.5 and if we have estimated it to be 3.0, then we would have lost an availability of 0.000523 which can be considered negligible.

Table 4.6 shows the values of availability for  $\alpha = 3.5$ ,  $\mu = 1390$ ,  $R_r = 1$  hour,  $R_p = 8$  hours. The optimal replacement age for the two methods and the maximum availability are:

- approximate solution      1935 hours      0.994243
- exact solution              1940 hours      0.994253

Once again it is observed that the approximation gives very accurate results. If the actual shape parameter was 2.5 and if we have estimated it to be 3.5, then we would have lost an availability of 0.000859 which again can be considered negligible. This shows that the shape parameter does not effect the availability drastically and a close estimate is sufficient.

Table 4.7 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1350$ ,  $R_r = 1$  hour,  $R_p = 8$  hours. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	2140 hours	0.994421
• exact solution	2145 hours	0.994430

It is observed that the approximation gives very accurate results. If the actual scale parameter was 1390 and if we have estimated it to be 1350, then we would have lost an availability of 0.000159 which again can be considered negligible.

Table 4.8 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1450$ ,  $R_r = 1$  hour,  $R_p = 8$  hours. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	2298 hours	0.994805
• exact solution	2302 hours	0.994812

It is observed that the approximation gives very accurate results. If the actual scale parameter was 1450 and if we have estimated it to be 1350, then we would have lost an availability of 0.000382 which again can be considered negligible. This shows that the scale parameter does not effect the availability drastically and a close estimate is sufficient.

**TABLE 4.3 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_T = 1$  HOUR,  $R_P = 8$  HOURS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1200.0	0.992845	0.992847
1300.0	0.993258	0.993261
1400.0	0.993593	0.993595
1500.0	0.993862	0.993865
1600.0	0.994076	0.994080
1700.0	0.994245	0.994249
1800.0	0.994374	0.994379
1900.0	0.994469	0.994474
2000.0	0.994532	0.994539
2100.0	0.994569	0.994576
2200.0	0.994581	0.994589
2300.0	0.994571	0.994580
2400.0	0.994540	0.994551
2500.0	0.994490	0.994503
2600.0	0.994423	0.994437

**TABLE 4.4 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_T = 8$  HOURS,  $R_P = 8$  HOURS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
800.0	0.988211	0.988273
900.0	0.988798	0.988870
1000.0	0.989108	0.989193
1100.0	0.989201	0.989300
1200.0	0.989116	0.989232
1300.0	0.988880	0.989014
1400.0	0.988513	0.988669
1500.0	0.988028	0.988210
1600.0	0.987437	0.987648
1700.0	0.986748	0.986993
1800.0	0.985967	0.986252
2000.0	0.984148	0.984530
2200.0	0.982012	0.982519
2400.0	0.979577	0.980244
2600.0	0.976857	0.977724

**TABLE 4.5 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 2.5$ ,  $\mu = 1390$ ,  $R_r = 1$  HOUR,  $R_p = 8$  HOURS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
2000.0	0.994779	0.994783
2100.0	0.994874	0.994878
2200.0	0.994949	0.994954
2300.0	0.995008	0.995013
2400.0	0.995051	0.995057
2500.0	0.995080	0.995087
2600.0	0.995098	0.995105
2700.0	0.995104	0.995112
2800.0	0.995100	0.995108
2900.0	0.995087	0.995096
3000.0	0.995065	0.995075
3100.0	0.995036	0.995047
3200.0	0.995000	0.995011
3300.0	0.994956	0.994969
3400.0	0.994907	0.994921

**TABLE 4.6 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 8.5$ ,  $\mu = 1890$ ,  $R_p = 1$  HOUR,  $R_r = 8$  HOURS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1200.0	0.992883	0.992885
1300.0	0.993279	0.993281
1400.0	0.993590	0.993593
1500.0	0.993829	0.993833
1600.0	0.994007	0.994012
1700.0	0.994132	0.994137
1800.0	0.994208	0.994215
1900.0	0.994242	0.994250
2000.0	0.994236	0.994246
2100.0	0.994194	0.994206
2200.0	0.994118	0.994132
2300.0	0.994009	0.994026
2400.0	0.993869	0.993889
2500.0	0.993699	0.993723
2600.0	0.993501	0.993529

**TABLE 4.7 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1350$ ,  $R_T = 1$  HOUR,  $R_P = 8$  HOURS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1300.0	0.993201	0.993204
1400.0	0.993526	0.993529
1500.0	0.993785	0.993789
1600.0	0.993990	0.993993
1700.0	0.994147	0.994152
1800.0	0.994264	0.994269
1900.0	0.994346	0.994352
2000.0	0.994397	0.994404
2100.0	0.994419	0.994428
2200.0	0.994417	0.994426
2300.0	0.994391	0.994402
2400.0	0.994344	0.994357
2500.0	0.994278	0.994293
2600.0	0.994193	0.994210
2700.0	0.994092	0.994110

**TABLE 4.8 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1450$ ,  $R_F = 1$  HOUR,  $R_P = 8$  HOURS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1500.0	0.993961	0.993963
1600.0	0.994189	0.994192
1700.0	0.994373	0.994376
1800.0	0.994517	0.994521
1900.0	0.994628	0.994632
2000.0	0.994709	0.994714
2100.0	0.994764	0.994770
2200.0	0.994795	0.994802
2300.0	0.994805	0.994812
2400.0	0.994795	0.994803
2500.0	0.994767	0.994777
2600.0	0.994722	0.994733
2700.0	0.994662	0.994674
2800.0	0.994587	0.994601
2900.0	0.994498	0.994514



## V. POLICY III (MINIMAL REPAIR / FAILURE REPLACEMENT / PREVENTIVE REPLACEMENT)

POLICY II assumes that each time a failure occurs it can be repaired and the system is restored to an operational state without changing the failure rate of the system as a whole. Now we shall be more realistic and consider the possibility of a major failure before the planned age replacement time  $t_p$ , and that failure is rectified by a replacement. (Here again a replacement may be an overhaul which is assumed to return the system to a state as good as new). So when the system is running, two types of failures are possible:

- Type I failure, denoted by 1C. This failure is *corrected* by minimal repair; if the Type I failure occurs at age  $t$ , the failure rate just after correction is  $h(t)$ . This type of failure is usually repaired at sea by the ship's staff using the support elements onboard.
- Type II failure denoted by 2C. This failure is remedied by effective system replacement; the Type II failure is followed by overhaul, after which the failure rate is  $h(0)$ . This type of failure is beyond the ship's staff capability either due to lack of expertise or unavailability of the required support elements, and the ship has to return to base to effect repair either by major overhaul or replacement.

The failures are detected immediately.

It is assumed that the costs and downtime for simple preventive maintenance actions are negligible.

First let us assume the time to some type of failure,  $T$ , has an IFR distribution  $F(t)$  with a failure rate denoted by  $h(t)$ ;  $0 \leq t$  is the system operating time since last overhaul. Let  $Y$  be the time of a type 2C failure having the distribution function  $G(t)$ .

Let  $p_1$  be the probability of Type 1C failure

and  $p_2$  be the probability of Type 2C failure

( $p_1$  and  $p_2$  could dependent on time) ;  $p_1 + p_2 = 1.0$

Now let  $N(t)$  denote the number of Type 1C failures that occurs in time  $t$ , where  $t$  is measured from a moment of replacement or overhaul. By the assumption, the expected number of Type 1C failures in time  $t$ ,  $m(t) = E[N(t)]$ , satisfies a simple differential equation obtained as follows:

$$m(t+dt) = m(t) \{1 - p_1 h(t) dt\} + \{m(t)+1\} p_1 h(t) dt + o(dt) \quad (5.1)$$

where  $o(dt)$  represents higher order terms which are  $\approx 0$

rearranging the terms we get

$$\frac{dm(t)}{dt} = p_1 h(t) \quad (5.2)$$

and therefore

$$m(t) = p_1 \int_0^t h(t_1) dt_1 \quad (5.3)$$

*provided* minimal repair correction times are assumed to be negligible enough so that the expected downtime from that source is close to  $R_{1C} \cdot m(t)$ , where  $R_{1C}$  is the mean time of a minimal repair.

Let  $T$  be the time to a failure (either Type 1C or 2C) with the failure rate  $h(t)$

Then the failure rate of Type I failure =  $p_1 h(t)$  at age  $t$

and the failure rate of Type II failure =  $p_2 h(t)$  at age  $t$ .

Since

$$[1 - F(t)] = e^{-\int_0^t h(t) dt} \quad (5.4)$$

and

$$\begin{aligned} [1 - G(t)] &= e^{-\int_0^t p_2 h(t) dt} \\ &= \{e^{-\int_0^t h(t) dt}\}^{p_2} \end{aligned}$$

$$\therefore [1 - G(t)] = [1 - F(t)]^{p_2} \quad (5.5)$$

From equation (5.5), it can be seen that if  $p_2 = 1$ , then the distribution of any type of failure is the same as the distribution of a Type 2C failure and this implies that each failure is a major failure and it requires a replacement / overhaul action and this is the case as in Policy I (Age Replacement) as mentioned in Chapter III.

For  $p_2 = 0$ , this implies that the probability of a major failure, i.e Type 2C is 0 and each failure is a minimal failure and the failure is removed by

minimal repair and this is the case as in Policy II (Minimal Repair and Preventive Replacement) as mentioned in Chapter IV.

For the case when  $0 < p_2 < 1$ , then a failure could be either

- Type 1C
- Type 2C

This is Policy III, which is the general case and will be discussed in this chapter.

This idea of two types of failure has been studied by Beichelt and Fisher [Ref. 18]. They derived the Reliability functions for calculating the expected long run cost rate for a generalised age-replacement policy. They assumed that maintenance actions take only negligible times which in reality is not true, especially when a major failure occurs at sea. Besides that, in the military environment we are often interested in the availability of the system as our measure of effectiveness so in this policy we will expand the cost rate model, but also, and more importantly, formulate a model to maximize the availability of the system subjected to two types of failures.

#### **A. COST MODEL**

The age  $t_p$  at which the operating system is replaced or overhauled depends on the following factors:

- Failure distribution (IFR).
- Costs of minimal repair, preventive replacement and failure replacement.
- Downtime of preventive replacement and failure replacement.
- Measure of effectiveness:
  - Minimize costs.
  - Maximize availability.

It is assumed that the downtime of a minimal repair is negligible.

Let  $C_{1c}$  be the cost of a minimal repair.

$C_{2c}$  be the cost of a failure replacement.

$C_{2p}$  be the cost of a preventive replacement.

$R_{1c}$  be the mean time of a minimal repair (Type 1C failure).

$R_{2c}$  be the mean time of a failure replacement (Type 2C failure).

$R_{2p}$  be the mean time of a preventive replacement.  $t_p$  be the planned replacement age.

$N(t_p)$  be the number of minimal repairs in interval  $(0, t_p)$ .

$y$  be the observed time from system replacement until the next Type 2C failure.

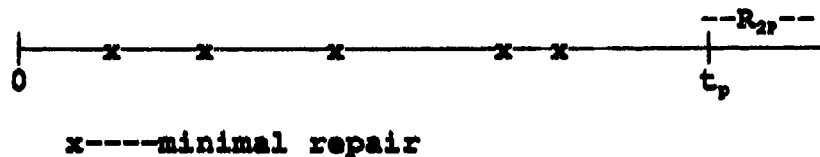
$N(y)$  be the number of minimal repairs in the period  $(0, y)$ , where  $y$  is the time until Type 2C failure.

$$R_{2c} > R_{2p} > R_{1c} \quad \text{and} \quad C_{2c} > C_{2p} > C_{1c}$$

A replacement takes place either at time  $t_p$  or when there is a Type 2C failure.

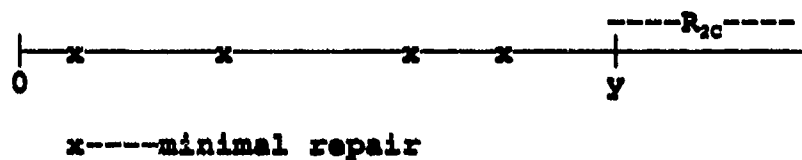
1. Case 1

$$Y > t_p$$



2. Case 2

$$Y < t_p$$



A cycle is completed each time a replacement takes place and the costs  $C$  incurred in a cycle is given by the total costs of minimal repairs and the cost of either a preventive replacement or a failure replacement. This is given by the following:

$$C = \begin{cases} C_{1c} [ N(t_p) \mid Y > t_p ] + C_{1c} & \text{w.p. } 1 - G(y) \\ C_{1c} [ N(y) \mid Y \leq t_p ] + C_{2c} & \text{w.p. } G(y) \end{cases} \quad (5.6)$$

In order to find the total costs of minimal repairs, we ought to know the expected number of minimal repairs and this is given below for both the two cases:

For case 1,  $Y > t_p$

$$E[N(t_p) | Y > t_p] = \int_0^{t_p} p_1 h(t) dt \quad (5.7)$$

For case 2,  $Y \leq t_p$

Let  $N(y) = n$ , then

$$\begin{aligned} E[N(y) | Y \leq t_p] &= \sum_{n=0}^{\infty} n P[N(y)=n | Y \leq t_p] \\ &= \sum_{n=0}^{\infty} \frac{n P[N(y)=n, Y \leq t_p]}{P[Y \leq t_p]} \end{aligned} \quad (5.8)$$

thus

$$E[N(y) | Y \leq t_p] = \frac{1}{G(t_p)} \int_0^{t_p} \left[ \int_0^t p_1 h(s) ds \right] g(t) dt \quad (5.9)$$

Then

$$E[C] = [(C_{1c} E[N(t_p) | Y > t_p]) + C_{2p}] [1 - G(t_p)] + [(C_{1c} E[N(y) | Y \leq t_p]) + C_{2c}] [G(t_p)] \quad (5.10)$$

The length of the cycle,  $L$ , depends on the time of a Type 2C failure and is given by:

$$L = \begin{cases} R_{2p} + t_p & Y > t_p \\ R_{2c} + Y & Y \leq t_p \end{cases} \quad (5.11)$$

Then the expected length of the cycle,  $E[L]$ , is

$$E[L] = R_{2p}[1 - G(t_p)] + R_{2c}[G(t_p)] + M(t_p) \quad (5.12)$$

where  $M(t_p)$  is the mean age of the system at replacement / overhaul and is given by

$$M(t_p) = \int_0^{t_p} t g(t) dt + t_p[1 - G(t_p)] \quad (5.13)$$

which can be simplified to

$$M(t_p) = \int_0^{t_p} [1 - G(t)] dt \quad (5.14)$$

From equation (3.1),

$$C(t_p) = \frac{E[C]}{E[L]}$$

Then the optimal value of  $t_p^*$  is the value that minimizes the cost rate function  $C(t_p)$  and this can be found by graphical or numerical analysis. [Ref. 16] highlights that an finite optimal interval  $t_p^*$  exists if  $h(t)$  is monotone increasing function and  $C > 0$  where  $C$  is given by

$$C = \frac{C_{1c}}{(C_{2c} - C_{2p} - C_{1c})}$$

### 3. Weibull Example

For a time to failure,  $T$ , following a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$  the expected total cost in a cycle,  $E[C]$ , is given as follows:



$$E[C] = e^{-P_2(\lambda t_p)^\alpha} \{C_{1C} p_1 (\lambda t_p)^\alpha + C_{2P} - C_{2C}\} + C_{2C} \quad (5.15)$$

$$C_{1C} p_1 p_2 \propto \lambda^{2\alpha} \int_0^{t_p} t^{2\alpha-1} e^{-P_2(\lambda t)^\alpha} dt$$

The integral  $\int_0^{t_p} t^{2\alpha-1} e^{-P_2(\lambda t)^\alpha} dt$  has no closed form

solution and the IMSL subroutine DQDAG was used to solve it numerically.

The length of the cycle is given by

$$E[L] = R_{2P}\{e^{-P_2(\lambda t_p)^\alpha}\} + R_{2C}\{1 - e^{-P_2(\lambda t_p)^\alpha}\} + \int_0^{t_p} e^{-P_2(\lambda t)^\alpha} dt \quad (5.16)$$

#### 4. Numerical Illustration

Now let us consider an example with these data: assuming that the downtime for a minimal repair is negligible:

$$C_{2P} = \$ 25,000$$

$$C_{2C} = \$ 37,500$$

$$C_{1C} = \$ 1,000$$

$$R_{2P} = 8 \text{ hours}$$

$$R_{2C} = 16 \text{ hours}$$

$$R_{1C} = 1 \text{ hour}$$

$$\alpha = 3.0$$

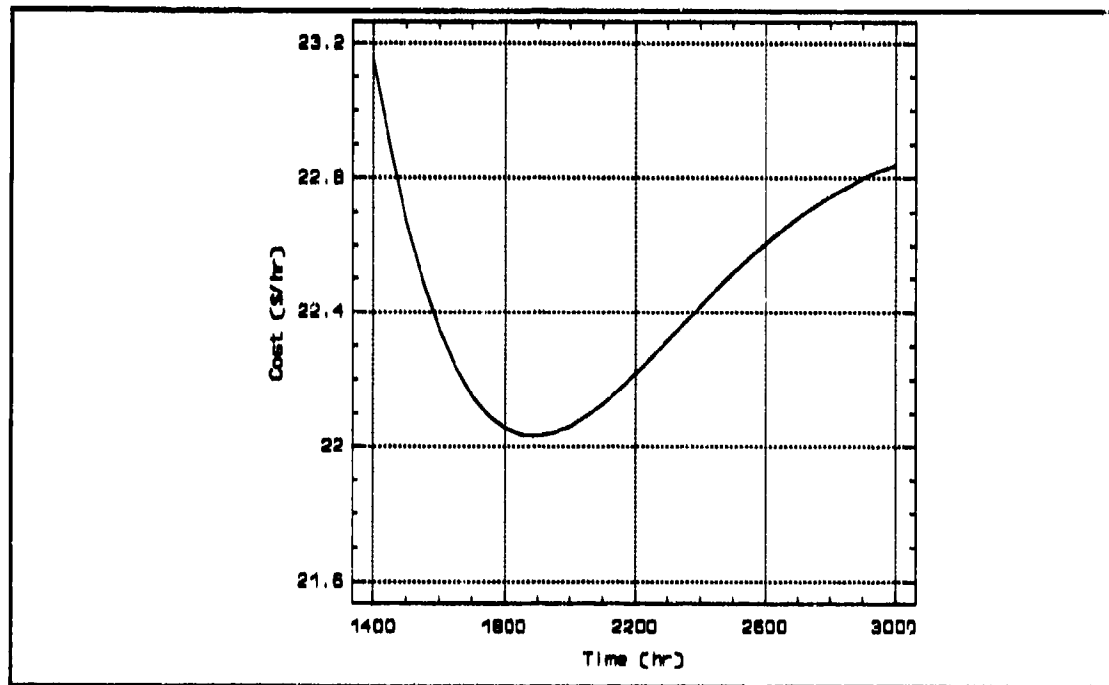
$$\lambda = 1/1390 \text{ hours}$$

$$p_1 = 0.6$$

$$p_2 = 0.4$$

Figure 5.1 shows the plot of the expected cost rate function. It is observed that the curve is fairly flat near the optimal point thus giving flexibility to the decision maker. The optimal replacement age is  $t_p = 1888.64$  hours at a cost of \$22.03 per hour.

In a real situation  $p_2$  is a function of age and it usually increases with age i.e. as the system ages the probability of a major failure approaches 1.0. For our case we have taken the probability to be constant and this again is to simplify our computation.



**Figure 5.1 Plot of the Cost Rate Function.**

**TABLE 5.1 VALUES OF COST/HOUR FOR VARIOUS REPLACEMENT AGE.**

<b>Replacement age <math>t_p</math> (hours)</b>	<b>Cost (\$ / hour)</b>
1000.0	27.67
1100.0	26.01
1200.0	24.76
1300.0	23.83
1400.0	23.15
1500.0	22.67
1600.0	22.35
1700.0	22.15
1800.0	22.06
1900.0	22.03
2000.0	22.06
2100.0	22.13
2200.0	22.22
2300.0	22.32
2400.0	22.42
2500.0	22.52

## B. AVAILABILITY MODEL

Now let us take the (Point) Availability as our measure of effectiveness. When the ship is at sea there are usually two types of failures; failures which can be rectified onboard, each of which individually does not effect the availability of the system drastically, and other failures which are not repairable onboard so the ship has to return to port to effect repairs; this causes the availability to be more severely degraded. First we shall assume that the downtime for a minimal repair is negligible. Then using the equation (3.8), the availability function is given by

$$A(t_p) = \frac{M(t_p)}{R_{2p} [1 - G(t_p)] + R_{2c} [G(t_p)] + M(t_p)} \quad (5.17)$$

where  $M(t_p)$  is the mean operational time in a cycle and it is given by

$$M(t_p) = \int_0^{t_p} [1 - G(t)] dt \quad (5.18)$$

and this is similar to the availability function in Policy I in Chapter III.

Again the availability function in equation (5.17) is an approximation because we do not take into account the downtime during a minimal repair, which in reality is not faithful to reality, but should be reasonably accurate.

If we do take the downtime of the minimal repair into account we can obtain an exact solution as follows:

Using the same principle as in equation (4.15),

Let

$Y$  = Time of a Major Failure

$X(t) = \begin{cases} 1 & \text{if the system is up at time } t \\ 0 & \text{if the system is down at time } t \end{cases}$

$\beta$  =  $P(\text{Failure is Type 1C})$

$a_1(t) = P[Y > t, X(t) = 1]$

$a_0(t) = P[Y > t, X(t) = 0]$

$a_1(t) + a_0(t) = P[Y > t] = 1 - G(t)$

$Z(t) = \begin{cases} X(t) & \text{if } Y > t \\ 0 & \text{if } Y \leq t \end{cases}$

$h(t)$  = the failure rate of Type 1C failure, which is  $p_1 h(t)$

$\mu(t)$  = the repair rate of Type 1C failure, and for the Exponential distribution,  $\mu(t) = R_{1C}^{-1}$

Now,

$$a_1(t+dt) = a_1(t) \{1 - h(t)dt\} + a_0(t)\mu(t)dt + o(dt) \quad (5.19)$$

$$a_0(t+dt) = a_0(t) \{1 - \mu(t)dt\} + a_1(t)h(t)dt + o(dt) \quad (5.20)$$

Initial conditions:  $a_1(0) = 1$  ;  $a_0(0) = 0$

Equation (5.19) becomes

$$\frac{d(a_1)}{dt} = -h(t)a_1(t) + a_0(t)\mu(t) \quad (5.21)$$

$$\frac{d(a_0)}{dt} = -\mu(t)a_0(t) + a_1(t)h(t)\beta \quad (5.22)$$

Since  $a_0(t) = [1 - G(t)] - a_1(t)$ , Equation (5.21) becomes

$$\frac{d(a_1)}{dt} = -h(t)a_1(t) + \mu(t)\{[1 - G(t)] - a_1(t)\} \quad (5.23)$$

which can be simplified to

$$\frac{d(a_1)}{dt} = -\{h(t) + \mu(t)\}a_1(t) + \mu(t)\{1 - G(t)\} \quad (5.24)$$

The equation,

$$\frac{d(a_1)}{dt} + \{h(t) + \mu(t)\}a_1(t) = 0 \quad ; \quad \text{with } a_1(0) = 1$$

has the solution

$$a_1(t) = \text{EXP}\{-E(t)\}$$

where

$$E(t) = \int_0^t \{h(s) + \mu(s)\} ds$$

Therefore the equation (5.24) has the solution

$$a_1(t) = e^{-E(t)} [1 + \int_0^t \{\mu(s)\{1 - G(s)\} e^{-E(s)} ds\} \quad (5.25)$$

and equation (5.25) can now be written as

$$p_1(t) = e^{-\int_0^t (h(t_1) + \mu(t_1)) dt_1} + \int_0^t \mu(t_2) \{1 - G(t_2)\} e^{-\int_2^t (h(t_1) + \mu(t_1)) dt_1} dt_2 \quad (5.26)$$

and for the Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\lambda$ , and substituting the values for  $h(t)$  and  $\mu(t)$  in equation (5.26), the probability that the system is up at time  $t$  is given by

$$p_1(t) = e^{-[p_1(\lambda t)^\alpha + \frac{t}{R_{1c}}]} + \frac{1}{R_{1c}} \int_0^t e^{-[p_1(\lambda t)^\alpha - p_1(\lambda t_2)^\alpha + p_2(\lambda t_2)^\alpha + \frac{t}{R_{1c}} - \frac{t_2}{R_{1c}}]} dt_2 \quad (5.27)$$

But the integral  $\int_0^t e^{-[p_1(\lambda t)^\alpha - p_1(\lambda t_2)^\alpha + p_2(\lambda t_2)^\alpha + \frac{t}{R_{1c}} - \frac{t_2}{R_{1c}}]} dt_2$  has no closed-form

expression in terms of elementary tabulated functions. Therefore we have to evaluate it numerically.

Since the length of the cycle,  $L$  is

$$L = \begin{cases} R_{2r} + t_p & Y > t_p \\ R_{2c} + Y & Y \leq t_p \end{cases}$$

then using the same principle as that used in [Ref. 16] the average availability over a cycle of length  $L$  and hence in the long run is given by

$$A(t_p) = \frac{\int_0^{t_p} a_1(t) dt}{\text{Expected length of a Cycle}} \quad (5.28)$$

But the Expected length of a cycle is given by equation (5.12). Therefore the Availability,  $A(t_p)$ , is given by

$$A(t_p) = \frac{\int_0^{t_p} a_1(t) dt}{R_{2p}\{1 - G(t_p)\} + R_{2c}\{G(t_p)\} + \int_0^{t_p} 1 - G(t_p) dt} \quad (5.29)$$

To study the above expression the IMSL subroutine DQDAG was used to compute the Availability function  $a_1(t)$  for various values of  $t$ . Then we used the IMSL subroutine DCSINT [Ref. 13] to compute cubic spline interpolant to the set of data points obtained earlier (values of  $t$  and the corresponding values of  $a_1(t)$ ). Finally we used the IMSL subroutine DCSITG [Ref. 13] to evaluate the integral of the cubic spline for various values of the replacement age  $t_p$ . From this we can compute the optimal replacement age  $t_p^*$  and the maximum availability. The values obtained by using this 'exact' solution was compared with the approximation as in equation (5.17). These values are tabulated below. It is observed that both the approximate and exact solutions are the same to three decimal places, and the approximation gives an accurate solution. However this is true for small values of  $R_{1c}$  and  $R_{2c}$ . As the values of  $R_{1c}$  and  $R_{2c}$  are increased the accuracy of the present approximation method degrades.

Table 5.2 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_{1c} = 1$  hr,  $R_{2c} = 16$  hrs,  $R_{2p} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:



• approximate solution	1528 hours	0.991715
• exact solution	1496 hours	0.991567

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.991565, which is extremely close to the exact solution, and much more easily obtained.

Table 5.3 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  hr,  $R_{2C} = 24$  hrs,  $R_{2P} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	1201 hours	0.989796
• exact solution	1113 hours	0.989070

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.989015 which is extremely close to the exact solution.

Table 5.4 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  hr,  $R_{2C} = 72$  hrs,  $R_{2P} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	752 hours	0.984143
• exact solution	736 hours	0.983835

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.983827 which is very close to the exact solution.

Table 5.5 shows the values of availability for  $\alpha = 3.5$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  hr,  $R_{2C} = 24$  hrs,  $R_{2P} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	1147 hours	0.990119
• exact solution	1076 hours	0.989519

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.989471 which is again very close to the exact solution.

Table 5.6 shows the values of availability for  $\alpha = 2.5$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  hr,  $R_{2C} = 24$  hrs,  $R_{2P} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	1317 hours	0.989493
• exact solution	1200 hours	0.988594

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.988535 which is once again very close to the exact solution.

From the above results we can conclude that the approximate solution is a good approximation for planning the replacement or overhaul actions of a system in order to maximize the availability of the system. From our earlier

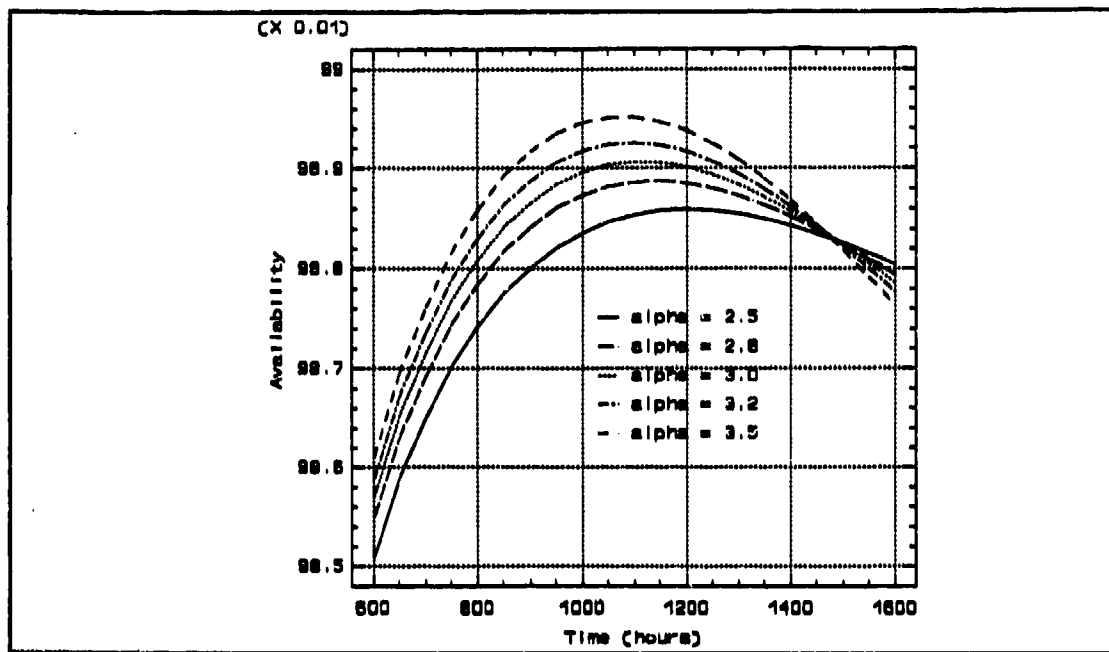
discussion we found that the availability is sensitive to the shape parameter  $\alpha$ ; Figure 5.2 shows the availability (exact solution) for various values of alpha.

If the shape parameter  $\alpha$  was in fact 3.5 and in our estimation we used  $\alpha = 2.5$ , then we would have lost an availability of 0.000626, which is extremely small. This shows that the parameters need not be estimated very accurately to achieve good results.

Figure 5.2 shows that for low values of alpha such as 2.5, the availability function is rather flat and there is more flexibility in determining the replacement age that is, the replacement interval at 1200 hours or 1500 hours gives about the same availability on the long run. However this is not true for higher values of alpha such as 3.5 where the availability function falls quite rapidly on both sides of the optimal replacement age  $t_p^*$ . At the availability of about 0.9883, the replacement age is 1475 hours and it is insensitive to the value of alpha.

Now we shall also look at the effects of the scale parameter  $\lambda$  on the availability. Table 5.7 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1350$ ,  $R_{1C} = 8$  hr,  $R_{2C} = 24$  hrs,  $R_{2P} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:

• approximate solution	1167 hours	0.989496
• exact solution	1081 hours	0.988750



**Figure 5.2 The Plot of 'Exact' Availability for Various Values of the Shape Parameter Alpha.**

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.988693 which is very close to the exact solution.

Now if the actual scale parameter was 1390 hours and it was estimated to be 1350 hours, then we would have lost an availability of 0.00032 which can be considered very small.

Table 5.8 shows the values of availability for  $\alpha = 3.0$ ,  $\mu = 1450$ ,  $R_{1C} = 8$  hr,  $R_{2C} = 24$  hrs,  $R_{2P} = 8$  hrs. The optimal replacement age for the two methods and the maximum availability are:

**TABLE 5.2 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_{1C} = 1$  HR,  $R_{2C} = 16$  HRS,  $R_{2P} = 8$  HRS**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1320.0	0.991592	0.991474
1344.0	0.991621	0.991499
1368.0	0.991646	0.991520
1392.0	0.991666	0.991537
1416.0	0.991683	0.991549
1440.0	0.991695	0.991559
1464.0	0.991705	0.991564
1488.0	0.991711	0.991567
1512.0	0.991715	0.991566
1536.0	0.991715	0.991563
1560.0	0.991713	0.991557
1584.0	0.991709	0.991549
1608.0	0.991702	0.991538
1632.0	0.991693	0.991525
1656.0	0.991682	0.991511

**TABLE 5.3 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_{1c} = 8$  HRS,  $R_{2c} = 24$  HRS,  $R_{2r} = 8$  HRS**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1018.0	0.989570	0.988997
1042.0	0.989629	0.989030
1066.0	0.989678	0.989053
1090.0	0.989718	0.989066
1114.0	0.989749	0.989070
1138.0	0.989772	0.989065
1162.0	0.989787	0.989052
1186.0	0.989794	0.989032
1210.0	0.989795	0.989004
1234.0	0.989789	0.988970
1258.0	0.989778	0.988929
1282.0	0.989760	0.988882
1306.0	0.989737	0.988830
1330.0	0.989709	0.988773
1354.0	0.989677	0.988710

**TABLE 5.4 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  HRS,  $R_{1C} = 72$  HRS,  $R_{1P} = 8$  HRS**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
520.0	0.982290	0.982140
544.0	0.982706	0.982542
568.0	0.983057	0.982877
592.0	0.983349	0.983154
616.0	0.983588	0.983376
640.0	0.983779	0.983550
664.0	0.983926	0.983680
688.0	0.984033	0.983768
712.0	0.984102	0.983819
736.0	0.984138	0.983835
760.0	0.984141	0.983818
784.0	0.984115	0.983772
808.0	0.984061	0.983697
832.0	0.983981	0.983596
856.0	0.983877	0.983470

**TABLE 5.5 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.5$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  HRS,  $R_{2C} = 24$  HRS,  $R_{2P} = 8$  HRS**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
944.0	0.989738	0.989330
968.0	0.989828	0.989394
992.0	0.989905	0.989445
1016.0	0.989969	0.989482
1040.0	0.990021	0.989506
1064.0	0.990061	0.989517
1088.0	0.990091	0.989517
1112.0	0.990110	0.989506
1136.0	0.990119	0.989484
1160.0	0.990119	0.989452
1184.0	0.990110	0.989411
1208.0	0.990092	0.989361
1232.0	0.990067	0.989302
1256.0	0.990034	0.989235
1280.0	0.989994	0.989160



**TABLE 5.6 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 2.5$ ,  $\mu = 1390$ ,  $R_{1C} = 8$  HRS,  $R_{2C} = 24$  HRS,  $R_{2P} = 8$  HRS**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1120.0	0.989330	0.988561
1144.0	0.989370	0.988579
1168.0	0.989404	0.988589
1192.0	0.989431	0.988594
1216.0	0.989454	0.988593
1240.0	0.989471	0.988587
1264.0	0.989482	0.988576
1288.0	0.989490	0.988560
1312.0	0.989493	0.988540
1336.0	0.989492	0.988515
1360.0	0.989487	0.988487
1384.0	0.989478	0.988456
1408.0	0.989466	0.988421
1432.0	0.989451	0.988383
1456.0	0.989433	0.988342

**TABLE 5.7 COMPARISON OF AVAILABILITY FOR VALUES OF**  
 $\alpha = 3.0$ ,  $\mu = 1350$ ,  $R_{1C} = 8$  HRS,  $R_{2C} = 24$  HRS,  $R_{2P} = 8$  HRS

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
948.0	0.989132	0.988588
972.0	0.989214	0.988643
996.0	0.989284	0.988687
1020.0	0.989343	0.988718
1044.0	0.989392	0.988738
1068.0	0.989430	0.988748
1092.0	0.989459	0.988749
1116.0	0.989480	0.988740
1140.0	0.989492	0.988722
1164.0	0.989496	0.988697
1188.0	0.989494	0.988664
1212.0	0.989484	0.988623
1236.0	0.989468	0.988577
1260.0	0.989446	0.988523
1284.0	0.989418	0.988464

**TABLE 5.8 COMPARISON OF AVAILABILITY FOR VALUES OF  
 $\alpha = 3.0$ ,  $\mu = 1450$ ,  $R_{1C} = 8$  HRS,  $R_{2C} = 24$  HRS,  $R_{2P} = 8$  HRS.**

<b>Replacement Age (hours)</b>	<b>Approximate Availability</b>	<b>Exact Availability</b>
1000.0	0.989812	0.989322
1024.0	0.989892	0.989379
1048.0	0.989961	0.989425
1072.0	0.990021	0.989461
1096.0	0.990072	0.989488
1120.0	0.990114	0.989506
1144.0	0.990148	0.989515
1168.0	0.990175	0.989516
1192.0	0.990194	0.989511
1216.0	0.990207	0.989497
1240.0	0.990213	0.989478
1264.0	0.990213	0.989452
1288.0	0.990208	0.989420
1312.0	0.990197	0.989382
1336.0	0.990181	0.989340

- approximate solution      1253 hours      0.990214
- exact solution              1161 hours      0.989516

If we use the approximate solution as our reference for the replacement action, the exact availability is 0.989464 which is again very close to the exact solution.

Now if the actual scale parameter was 1450 hours and it was estimated to be 1350 hours, then we would have lost an availability of 0.000446 which once again can be considered very small. This shows that the scale parameter does not effect the availability drastically and a close estimate is sufficient.

### **C. MAXIMIZE AVAILABILITY SUBJECT TO BUDGET CONSTRAINT**

The availability and cost are very important measures of effectiveness. We would like to have as many resources as possible to maximize the availability of a system, however in reality we are often limited by budget constraints. So we would like to achieve cost effectiveness, that is we would like to

maximize      Effectiveness Level

subject to      Budget  $\leq$  B

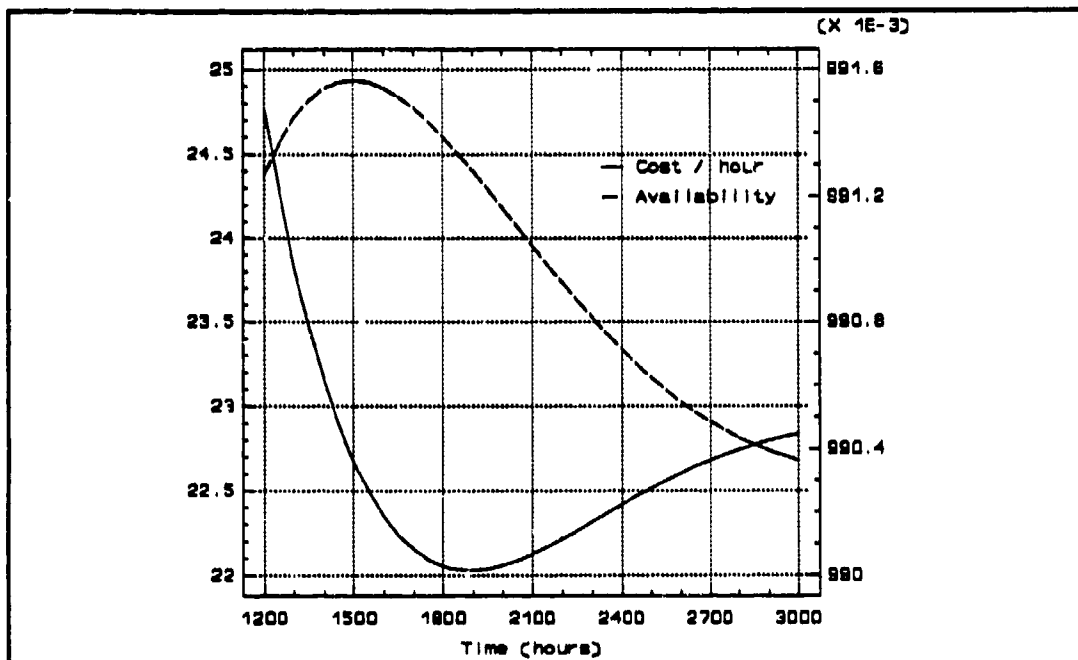
The effectiveness level which is commonly used is the availability. So in our case we would like to

Maximize       $A(t_p)$

Subject to       $C(t_p) \leq B.$

We have solved this problem graphically. Figure 5.3 shows the plots of the availability and the cost rate functions as formulated in Section B. It is observed that the availability is maximum at 0.991567 at an optimal replacement interval  $t_p^*$  of 1496 hours while the cost is minimum at \$22.03/hour at an optimal replacement interval  $t_p^*$  of 1889 hours.

Now if we have a budget of not exceeding \$22.25/hour, then the maximum availability that can be obtained is 0.9915 with an optimal replacement interval  $t_p^*$  of 1650 hours.



**Figure 5.3 Plot Showing the Availability and the Cost Rate Function.**

The other measure of effectiveness that is also used is the mission reliability and this again will give another optimal replacement age, so now we

have a multiple independent conflicting criterion and it is up to the Decision Maker to decide which of the measure of effectiveness is vital and fits the scenario very well.

## **VI. CONCLUSION AND RECOMMENDATIONS**

The fighting effectiveness and operational readiness of a ship depends largely on the operational availability of her equipment and systems. If we are not constrained by the budget then we can expend the resources necessary to achieve the desired availability, such as incorporating more redundancies or carrying out "premature" replacements and overhauls. However in reality this is not the case, and we are always limited by the available budget, and we would like to maximize system availability subject to budget constraints.

At sea, equipment and systems are exposed to various environment and unfavourable conditions. As such, they are subject to stochastic failure and deterioration. However, with timely maintenance actions as discussed in the various policies in the thesis, we can minimize catastrophic and unexpected failures, enabling us to achieve the desired measures of effectiveness. Therefore, based on the optimal maintenance policies, we can carry out replacement actions or complete overhaul of the equipments and systems at the base during the stand-off periods so that when the ship is at sea we can minimize loss of availability due to failures and maximize our successes in the operational missions.

Based on the policies, we can also carry adequate spares. This is particularly important for long missions or when the need to be prepositioned

in forward operating areas. On the other hand, although we can carry spares onboard, sometimes other support elements are not available to rectify the defects.

In our analysis, we have assumed that we know the failure distribution. For our case we concentrated on the Weibull distribution. The two important parameters which are usually estimated are the shape parameter,  $\alpha$ , and the scale parameter,  $\lambda$ . From the analysis it is observed that these parameters need not be estimated very accurately. A slight variation in the values of the parameters do not affect the long run availability and cost rate functions drastically. Of the two, the shape parameter needs to be estimated more accurately.

In the formulation of the availability functions, we simplified the computation by assuming the downtime for a minimal repair to be negligible. When compared with the exact solution, taking into account the downtime for a minimal repair, the results obtained by the approximation method gave extremely accurate results. Many of the functions and integrals that were formulated did not have closed form expressions in terms of elementary tabulated functions. However IMSL subroutines were available in the Math library at the main frame at the Naval Postgraduate School and these subroutines expeditiously computed the integrals very accurately.

The expected downtimes, as taken in our analysis, are practical figures assuming all the support elements are readily available when required. That



is the reason for the high values of the availability. In reality we often have to wait for spare parts and support elements, which sometimes have long lead times.

In the thesis we also studied the effects of simple preventive maintenance actions on the measures of effectiveness. It is observed that simple preventive maintenance actions do not restore the system to a condition "as good as new" but the maintenance actions can enhance or improve the reliability of the system by a certain factor which decreases as the system ages. However, a methodology for characterizing the effective age reduction remains to be developed. In our analysis we have assumed that the system improves by a certain factor on completion of each preventive maintenance action.

Most planned maintenance systems usually adopt maintenance efforts based on calendar time (weeks, months) or running hours of systems or equipment, but from the analysis it is observed that for systems that have a "wear-out" life distribution we shall have to successively resort to decreasing maintenance intervals if we are going to maintain the systems above some minimum reliability level.

This subject can be expanded further by future research. The following areas are recommended:

- Carry out similar analysis, especially for the availability function for the three policies when the underlying life distribution  $F$  comes from a family of Gamma Distribution.

- Incorporate imperfect repair into the model.
- A methodology for characterizing the improvement of the reliability of the system on completion of preventive maintenance needs to be investigated.
- A system may consist of various sub-systems. The maintenance actions for the sub-systems has to be coordinated so that instead of just taking into account the availability of the sub-systems individually, the whole system has to be considered bearing in mind of some dependence on the supporting elements associated with the sub-systems.

## APPENDIX A. COMPUTER PROGRAM AND OUTPUT

### PROGRAM PM

C THIS PROGRAM IS FORMULATED TO COMPUTE THE AVAILABILITY,  
C RELIABILITY AND COST FUNCTIONS FOR VARIOUS POLICIES OF  
C INTEREST. THE PROGRAM DETERMINES THE OPTIMAL REPLACEMENT  
C OR OVERHAUL INTERVALS FOR THE SYSTEM. THE OPTIMAL  
C REPLACEMENT INTERVAL DEPENDS ON THE MEASURES OF  
C EFFECTIVENESS DESIRED AND THE RELEVANT TYPE OF POLICY  
C APPLICABLE TO THE SYSTEM UNDER STUDY. THE PROGRAM  
C UTILIZES THE IMSL SUBROUTINES FROM THE MATH LIBRARY  
C AVAILABLE AT THE MAIN FRAME AT THE NAVAL POST GRADUATE  
C SCHOOL. IT IS ASSUMED THAT THE TIME TO FAILURE FOLLOWS  
C THE WEIBULL DISTRIBUTION WITH SHAPE PARAMETER ALPHA AND  
C SCALE PARAMETER LAMBDA.

C-----  
C **PARAMETER STATEMENT**

INTEGER NDATA,NTINT

PARAMETER (NDATA = 81, NTINT = 20)

C-----  
C **VARIABLES DECLARATION**

REAL\*8 A, B, F, RESULT, ERRABS, ERRREL, ERREST, ALPHA,  
& LAMBDA, MU, C2C, C1C, C2P, R2P, R2C, R1P, R1C,  
& A1, AA, XDATA(NDATA), YDATA(NDATA),  
& BREAK(NDATA), CSCOE(4,NDATA), AVAIL, AVAIL1,  
& P1, P2, SOLN, SF, CDF, LENGTH, G, H, ANSWER,  
& DCSITG, COST, COSMIN, ANS, D, REL, E, EE, FF,  
& TD2P, TDT1, TDT2, SUMD1P, SUMD2C, STIME, TL,  
& CDFTL, T(NTINT), REL, R, TDT, UPTIME, TD1P,  
& SUM, SUM1, X, XX, LL, UL, STEP

INTEGER IRULE,POL,P,SPMREQ,NN

C **VARIABLES DEFINITION**

C A.....LOWER LIMIT OF INTEGRATION USED IN  
C ARGUMENT OF THE IMSL SUBROUTINE DQDAG AND  
C IMSL FUNCTION DCSITG  
C B.....UPPER LIMIT OF INTEGRATION USED IN  
C ARGUMENT OF THE IMSL SUBROUTINE DQDAG AND  
C IMSL FUNCTION DCSITG  
C E,EE,F,FF,G,H..FUNCTIONS TO BE INTEGRATED  
C RESULT.....ESTIMATE OF THE INTEGRAL FROM A TO B OF  
C THE FUNCTIONS E,EE,F,FF  
C ERRABS.....ABSOLUTE ACCURACY DESIRED AS THE INPUT  
C ARGUMENT OF THE IMSL SUBROUTINES DQDAG  
C ERRREL.....RELATIVE ACCURACY DESIRED AS THE INPUT  
C ARGUMENT OF THE IMSL SUBROUTINES DQDAG  
C IRULE.....CHOICE OF QUADRATURE RULE (SEE APPENDIX  
C 2)  
C ALPHA.....SHAPE PARAMETER OF WEIBULL DISTRIBUTION  
C LAMBDA.....SCALE PARAMETER OF WEIBULL DISTRIBUTION  
C MU.....RECIPROCAL OF LAMBDA  
C C2C.....EXPECTED COST OF A FAILURE REPLACEMENT  
C C1C.....EXPECTED COST OF A MINIMAL REPAIR  
C C2P.....EXPECTED COST OF A PREVENTIVE REPLACEMENT  
C R2C.....EXPECTED DOWNTIME OF A FAILURE  
C REPLACEMENT  
C R1C.....EXPECTED DOWNTIME OF A MINIMAL REPAIR  
C R2P.....EXPECTED DOWNTIME OF A PREVENTIVE  
C REPLACEMENT  
C A1.....COMPUTE EXPONENTIAL EXPRESSION (POLICY  
C II)  
C AA.....AVAILABILITY AT TIME T (POLICY II)  
C NDATA.....NUMBER OF DATA POINTS FOR COMPUTING CUBIC  
C SPLINE INTERPOLANT. INPUT ARGUMENT FOR  
C IMSL SUBROUTINE DCSINT  
C XDATA( ).....ARRAY OF LENGTH NDATA CONTAINING THE DATA  
C POINTS ABSCISSAS. INPUT ARGUMENT FOR IMSL  
C SUBROUTINE DCSINT  
C YDATA( ).....ARRAY OF LENGTH NDATA CONTAINING THE DATA  
C POINTS ORDINATES. INPUT ARGUMENT FOR IMSL  
C SUBROUTINE DCSINT  
C BREAK( ).....ARRAY OF LENGTH NDATA CONTAINING THE  
C BREAKPOINTS FOR THE PIECEWISE CUBIC  
C REPRESENTATION. OUTPUT ARGUMENT OF THE  
C IMSL SUBROUTINE DCSINT  
C CSCOEFF.....MATRIX OF SIZE 4 BY NDATA CONTAINING THE  
C LOCAL COEFFICIENTS OF THE CUBIC PIECES.  
C OUTPUT ARGUMENT OF THE IMSL SUBROUTINE  
C DCSINT

C AVAIL.....APPROXIMATE AVAILABILITY  
 C ANSWER.....EXACT AVAILABILITY  
 C P1.....PROBABILITY OF TYPE I (MINIMAL) FAILURE  
 C P2.....PROBABILITY OF TYPE II (MAJOR) FAILURE  
 C SF.....SURVIVAL FUNCTION  
 C CDF.....DISTRIBUTION FUNCTION  
 C LENGTH.....LENGTH OF A CYCLE  
 C COST, COSMIN...COST RATE FUNCTION  
 C D.....MISSION DURATION  
 C REL.....RELIABILITY  
 C TD2P, TDT2.....TOTAL DOWNTIME OF PREVENTIVE REPLACEMENT  
 C TD1P, TDT1.....TOTAL DOWNTIME OF PREVENTIVE MAINTENANCE  
 C SUMD1P .....TOTAL DOWNTIME WHEN TIME OF FAILURE IS  
 C AFTER PREVENTIVE REPLACEMENT  
 C SUMD2C .....TOTAL DOWNTIME WHEN TIME OF FAILURE IS  
 C BEFORE PREVENTIVE REPLACEMENT  
 C STIME .....TOTAL DOWNTIME IN A CYCLE  
 C UPTIME .....TOTAL UPTIME IN A CYCLE  
 C LL .....LOWER LIMIT OF REPLACEMENT INTERVAL  
 C UL .....UPPER LIMIT OF REPLACEMENT INTERVAL  
 C STEP.....STEP SIZE OR INCREMENT  
 C POL.....POLICY TO BE EVALUATED  
 C SPMREQ.....SIMPLE PREVENTIVE MAINTENANCE REQUIREMENT

C-----  
 C **EXTERNAL SUBROUTINES AND FUNCTIONS**

EXTERNAL E,EE,F,FF,G,H,DQDAG,DCSINT,DCSITG

C-----  
 C **COMMON BLOCKS**

COMMON /FT/ R1C,B  
 COMMON /GT/ P2  
 COMMON /IT/ P1  
 COMMON /HT/ LAMBDA,ALPHA

C-----  
 C **INITIALIZATION**

A = 0.0D0  
 ERRABS = 0.0D0  
 ERRREL = 0.00001D0  
 IRULE = 2  
 PREL = 1.0D0  
 SUMD1P = 0.0D0

SUMD2C = 0.0D0  
STIME = 0.0D0  
SUM = 0.0D0

C-----  
C Prompt the user for the policy to be evaluated

C POLICY I ----- AGE REPLACEMENT  
C POLICY II ----- AGE REPLACEMENT WITH MINIMAL REPAIR  
C POLICY III ----- AGE REPLACEMENT WITH TWO TYPES OF  
C FAILURES. TYPE I MINIMAL FAILURE, TYPE  
C II MAJOR FAILURE

PRINT\*, 'ENTER THE NUMBER OF THE POLICY TO BE EVALUATED'  
READ\*, POL  
PRINT\*, 'POLICY = ', POL  
GOTO (100, 200, 300), POL

100 CONTINUE

PRINT\*, 'DO YOU WANT TO INCLUDE SIMPLE PREVENTIVE  
& MAINTENANCE-ENTER 1 FOR NO AND 2 FOR YES'  
READ\*, SPMREQ  
GOTO (125, 150), SPMREQ

C-----  
C POLICY I (AGE REPLACEMENT-NO SIMPLE PREVENTIVE  
C MAINTENANCE)  
C-----

125 CONTINUE

PRINT\*, 'ENTER THE ESTIMATED VALUES FOR THE SHAPE  
& PARAMETER ALPHA AND THE SCALE PARAMETER MU '  
READ\*, ALPHA, MU  
PRINT\*, 'ALPHA = ', ALPHA, 'MU = ', MU

PRINT\*, 'ENTER ESTIMATED VALUES FOR THE EXPECTED COST OF  
& PREVENTIVE REPLACEMENT FOLLOWED BY THE EXPECTED  
& COST OF FAILURE REPLACEMENT'  
READ\*, C2P, C2C  
PRINT\*, 'EXPECTED COST OF A PREVENTIVE REPLACEMENT =  
& ', C2P  
& ', EXPECTED COST OF A FAILURE REPLACEMENT =  
& ', C2C

PRINT\*, 'ENTER VALUES FOR THE EXPECTED DOWNTIME OF

```

&    PREVENTIVE REPLACEMENT FOLLOWED BY EXPECTED
&    DOWNTIME OF A FAILURE REPLACEMENT
READ*,R2P,R2C
PRINT*,'EXPECTED DOWNTIME OF A PREVENTIVE REPLACEMENT =
&                                     ',R2P
&    , 'EXPECTED DOWNTIME OF A FAILURE REPLACEMENT  =
&                                     ',R2C

PRINT*,'ENTER THE VALUE OF MISSION DURATION TIME IN
&    HOURS'
READ*,D
PRINT*,' MISSION DURATION TIME = ', D, 'HOURS'

PRINT*,'ENTER THE VALUE FOR LOWER LIMIT, UPPER LIMIT,
&    STEP SIZE '
READ*,LL,UL,STEP
PRINT*,'LOWER LIMIT = ',LL,'UPPER LIMIT = ',UL,'STEP =
&    ',STEP

WRITE(30,101)ALPHA,MU,C2P,C2C,R2P,R2C,D
101 FORMAT(15X,'POLICY',3X,'T',3X,'(AGE REPLACEMENT)'
&    ,/,15X,30('-')//,
&    10X,'ALPHA = ',F3.1,17X,    'MU = ',F6.1/,
&    10X,'C2P = ',F7.1,12X,    'C2C = ',F7.1/,
&    10X,'R2P = ',F4.1,'HOURS',11X,'R2C = ',F4.1,'
&    HOURS
&    ',/,10X,'MISSION DURATION = ',F4.1,' HOURS'//,
&    5X,'REPLACEMENT',5X,'COST RATE',5X,'AVAILABILITY',5X,
&    'MISSION RELIABILITY',
&    5X,'AGE (HOURS)',5X,'($/HOUR)',10X,'(A)', 17X,'(R)'/,
&    5X,11('-'),    5X,8('-'),    6X,12('-'), 5X,19('-')

LAMBDA = 1.0D0/MU
DO 11 I = LL,UL,STEP
    B=DBLE(I)
    CALL DQDAG (E,A,B,ERRABS,ERRREL,IRULE,RESULT,ERREST)
    SF=EXP(-((LAMBDA*B)**ALPHA))
    CDF=1.0D0-SF
    COST=((C2P*SF)+(C2C*CDF))/((R2P*SF)+(R2C*CDF)+RESULT)
    AVAIL=RESULT/((R2P*SF)+(R2C*CDF)+RESULT)
    REL= DEXP(-((LAMBDA*(B+D)**ALPHA)) /
&    DEXP(-((LAMBDA*B)**ALPHA))

WRITE(30,102)B,COST,AVAIL,REL
102  FORMAT(5X,F7.1,9X,F8.3,8X,F8.6,12X,F8.6)

```

11 CONTINUE

GOTO 99

```
C-----
C POLICY I (AGE REPLACEMENT - WITH SIMPLE PREVENTIVE
C MAINTENANCE)
C-----
```

150 CONTINUE

```
PRINT*, 'ENTER THE ESTIMATED VALUES FOR THE SHAPE
& PARAMETER ALPHA AND THE SCALE PARAMETER MU '
READ*, ALPHA, MU
PRINT*, 'ALPHA = ', ALPHA, 'MU = ', MU
```

```
PRINT*, 'ENTER VALUES FOR THE EXPECTED DOWNTIME OF
& PREVENTIVE REPLACEMENT FOLLOWED BY EXPECTED
& DOWNTIME OF A FAILURE REPLACEMENT'
READ*, R2P, R2C
PRINT*, 'EXPECTED DOWNTIME OF A PREVENTIVE REPLACEMENT =
& ', R2P
& , 'EXPECTED DOWNTIME OF A FAILURE REPLACEMENT =
& ', R2C
```

```
PRINT*, 'ENTER THE EXPECTED DOWNTIME FOR SIMPLE
& PREVENTIVE MAINTENANCE AND THE IMPROVEMENT
& FACTOR ON COMPLETION OF THE MAINTENANCE'
READ*, R1P, R
PRINT*, 'THE DOWNTIME FOR SIMPLE PREVENTIVE MAINTENANCE
& = ', R1P
& , 'THE IMPROVEMENT FACTOR = ', R
```

```
PRINT*, 'ENTER THE VALUE FOR LOWER LIMIT, UPPER LIMIT,
& STEP SIZE'
READ*, LL, UL, STEP
PRINT*, 'LOWER LIMIT = ', LL, 'UPPER LIMIT = ', UL, 'STEP =
& ', STEP
```

```
WRITE(30,103) ALPHA, MU, R2P, R2C, R1P, R
103 FORMAT(5X, 'POLICY', 2X, 'I', 2X, '(AGE REPLACEMENT WITH
& SIMPLE', 1X, 'PREVENTIVE MAINTENANCE)', /, 8X,
& 63 (' '), //,
& 10X, 'ALPHA = ', F3.1, 17X, 'MU = ', F6.1, /,
& 10X, 'R2P = ', F4.1, ' HOURS', 11X, 'R2C = ', F4.1, /)
```



```

& HOURS',/,10X,R1P  = ',F4.1,' HOUR',12X,R  =
& ',F4.1,/,
& 5X,NO. OF INTERVAL',5X,REPLACEMENT AGE (HOURS)',5X,
& 'AVAILABILITY',5X,15('-'),5X,23('-'),5X,12('-')

LAMBDA = 1.0D0/MU
T(1) = (1.0D0/LAMBDA)*((-LOG(0.75D0)))**
& (1.0D0/ALPHA))
CDF = 1.0D0 - EXP(-((LAMBDA*T(1))**ALPHA))
SF = 1.0D0-CDF
B = T(1)

DO 10 I = 1,NTINT
  IF (I .GT. 1) THEN
    T(I) = T(I-1)+(((1.0D0-R)**(I-1))*T(1))
  END IF
10 CONTINUE

DO 20 N = LL,UL,STEP
  IF (N .EQ. 1) THEN
    TDT = (R2P*SF)+(R2C*CDF)
    A = 0.0D0
    CALL DQDAG(FF,A,B,ERRABS,ERRREL,IRULE,RESULT,
& ERREST)
    UPTIME = (ALPHA*(LAMBDA**ALPHA)*RESULT)+(SF*T(1))
  ELSE
    DO 30 J = 1,N
      IF (J .EQ. 1) THEN
        TL = 0.0D0
      ELSE
        TL = T(1)-(T(J)-T(J-1))
      END IF
      CDFTL = 1-EXP(-((LAMBDA*TL)**ALPHA))
      REL = 1-(CDF-CDFTL)
      PREL = PREL*REL
30 CONTINUE

    TD1P = (N-1)*R1P*PREL
    TD2P = R2P*PREL
    TDT1 = TD1P+TD2P
    PREL = 1.0D0

    DO 40 K = 2,N
      DO 50 L = 1,K-1
        IF (L .EQ. 1) THEN
          TL = 0.0D0

```

```

ELSE
  TL = T(1)-(T(L)-T(L-1))
END IF
CDFTL = 1-EXP(-((LAMBDA*TL)**ALPHA))
REL = 1-(CDF-CDFTL)
PREL = PREL*REL
50 CONTINUE

TL = T(1)-(T(K)-T(K-1))
CDFTL = 1-EXP(-((LAMBDA*TL)**ALPHA))
FT = CDF-CDFTL
SUMD1P = SUMD1P+((K-1)*R1P*PREL*FT)
SUMD2C = SUMD2C+(R2C*PREL*FT)
PREL = 1.0D0
40 CONTINUE

TDT2 = SUMD1P+SUMD2C+(R2C*CDF)
TDT = TDT1+TDT2
PREL = 1.0D0
DO 60 M = 2,N
  DO 70 P = 1,M-1
    IF (P.EQ. 1) THEN
      TL = 0.0D0
    ELSE
      TL = T(1)-(T(P)-T(P-1))
    END IF
    CDFTL = 1-EXP(-((LAMBDA*TL)**ALPHA))
    REL = 1-(CDF-CDFTL)
    PREL = PREL*REL
70 CONTINUE

    A = T(1)-(T(M)-T(M-1))
    CALL DQDAG(FF,A,B,ERRABS,ERRREL,IRULE,RESULT,
&             ERREST)
    ANSWER = ALPHA*(LAMBDA**ALPHA)*RESULT
    TL = T(1)-(T(M)-T(M-1))
    CDFTL = 1-EXP(-((LAMBDA*TL)**ALPHA))
    REL = 1-(CDF-CDFTL)
    ANSWER = ANSWER+(REL*(T(M)-T(M-1)))
    STIME = STIME +(PREL*ANSWER)
    PREL = 1.0D0
60 CONTINUE

    A = 0.0D0
    CALL DQDAG (FF,A,B,FRRABS,ERRREL,IRULE,
&              RESULT,ERREST)

```

```

      ANSWER = (ALPHA*(LAMBDA**ALPHA)*RESULT) +
&      (SF*T(1))
      UPTIME = STIME+ ANSWER
      END IF

      AVAIL = UPTIME/(UPTIME+TDT)
      WRITE(80,104)N,T(N),AVAIL
20  CONTINUE

104  FORMAT(10X,I3,19X,F6.1,17X,F8.6)
      GOTO 99

C-----
C  POLICY II (AGE REPLACEMENT - WITH MINIMAL REPAIR)
C-----

200  CONTINUE
      PRINT*,ENTER THE ESTIMATED VALUES FOR THE SHAPE
&      PARAMETER ALPHA AND THE SCALE PARAMETER MU '
      READ*,ALPHA,MU
      PRINT*,ALPHA =',ALPHA,'MU = ',MU

      PRINT*,ENTER ESTIMATED VALUES FOR THE EXPECTED COST OF
&      PREVENTIVE REPLACEMENT FOLLOWED BY THE EXPECTED
&      COST OF MINIMAL REPAIR'
      READ*,C2P,C1C
      PRINT*,EXPECTED COST OF A PREVENTIVE REPLACEMENT =
&      ',C2P
&      ',EXPECTED COST OF A MINIMAL REPAIR      =
&      ',C1C

      PRINT*,ENTER VALUES FOR THE EXPECTED DOWNTIME OF
&      PREVENTIVE REPLACEMENT FOLLOWED BY EXPECTED
&      DOWNTIME OF A MINIMAL REPAIR'
      READ*,R2P,R1C
      PRINT*,EXPECTED DOWNTIME OF A PREVENTIVE REPLACEMENT =
&      ',R2P
&      ',EXPECTED DOWNTIME OF A MINIMAL REPAIR      =
&      ',R1C

      PRINT*,ENTER THE VALUE FOR LOWER LIMIT, UPPER LIMIT,
&      STEP SIZE'
      READ*,LL,UL,STEP
      PRINT*,LOWER LIMIT = ',LL,'UPPER LIMIT = ',UL,'STEP =
&      ',STEP

```

LAMBDA = 1.0D0/MU

PRINT\*, 'DO YOU WANT TO INCLUDE SIMPLE PREVENTIVE  
& MAINTENANCE-TYPE 1 FOR NO AND 2 FOR YES'  
READ\*, SPMREQ

GOTO (225,250),SPMREQ

C-----  
C POLICY II (AGE REPLACEMENT - WITH MINIMAL REPAIR WITHOUT  
C SIMPLE PREVENTIVE MAINTENANCE)  
C-----

225 CONTINUE

WRITE(30,105)ALPHA,MU,C2P,C1C,R2P,R1C  
105 FORMAT(9X,'POLICY',2X,'IF',2X,'(AGE REPLACEMENT WITH  
& MINIMAL',1X,'REPAIR')',9X,50('-')//,  
& 10X,'ALPHA = ',F3.1,17X,'MU = ',F6.1/,  
& 10X,'C2P = ',F7.1,12X,'C1C = ',F7.1/,  
& 10X,'R2P = ',F4.1,' HOURS',11X,'R1C = ',F4.1,  
& HOURS'//,5X,'REPLACEMENT',6X,'COST',10X,'APPROXIMATE',  
& 8X,'EXACT'/, 5X,'AGE HOURS',5X,'\$/HOUR',9X, '  
& AVAILABILITY', 4X, 'AVAILABILITY',5X,11('-'),  
& 5X,7('-'), 8X,12('-'),4X,12('-'))

DO 12 I = 0,4000,50

B=DBLE(I)

CALL DQDAG (EE,A,B,ERRABS,ERRREL,IRULE,RESULT,ERREST)

A1 =DEXP(-(((LAMBDA\*B)\*\*ALPHA)+(B/R1C)))

AA = A1+(RESULT\*(1.0D0/R1C) )

WRITE(13,\*)B,AA

12 CONTINUE

REWIND (13)

DO 22 J = 1, NDATA

READ(13,\*)XDATA(J),YDATA(J)

22 CONTINUE

CALL DCSINT(NDATA,XDATA,YDATA,BREAK,CSCOEf)

C  
C Calculate the integral of the spline  
C approximation.

```

DO 32 K = LL,UL,STEP
  B = DBLE(K)
  NINTV = NDATA - 1
  ANSWER = DCSITG(A,B,NINTV,BREAK,CSCOE)
  ANSWER = ANSWER/(B+R2P)
  AVAIL = (B-(R1C*((LAMBDA*B)**ALPHA)))/(B+R2P)
  AVAIL1 = B/(B+R2P+(R1C*((LAMBDA*B)**ALPHA)))
  COST = ((C1C*((LAMBDA*B)**ALPHA))+C2P)/(B+R2P)
  WRITE (30,106) B,COST,AVAIL,ANSWER
32 CONTINUE
106 FORMAT(7X,F6.1,6X,F9.4,10X,F8.6,8X,F8.6)

```

GOTO 99

```

C-----
C POLICY II (AGE REPLACEMENT - WITH MINIMAL REPAIR AND
C SIMPLE PREVENTIVE MAINTENANCE)
C-----

```

250 CONTINUE

```

PRINT*, 'ENTER THE EXPECTED COST OF A SIMPLE PREVENTIVE
& MAINTENANCE FOLLOWED BY THE IMPROVEMENT IN THE
& AGE IN HOURS ACHIEVED ON COMPLETION OF THE
& MAINTENANCE'
READ*,C1P,XX
PRINT*, 'EXPECTED COST OF SIMPLE PREVENTIVE MAINTENANCE
& = ',C1P,
& IMPROVEMENT IN AGE(HOURS) = ',XX

PRINT*, 'ENTER THE NUMBER OF SIMPLE PREVENTIVE
& MAINTENANCE ACTIONS TO BE TAKEN IN A CYCLE '
READ*,NN
PRINT*, 'NUMBER OF SIMPLE PREVENTIVE MAINTENANCE ACTIONS
& = ',NN
PRINT*, 'ENTER THE VALUE FOR LOWER LIMIT, UPPER LIMIT,
& STEP SIZE'
READ*,LL,UL,STEP
PRINT*, 'LOWER LIMIT = ',LL, 'UPPER LIMIT = ',UL, 'STEP =
& ',STEP

WRITE(30,107)ALPHA,MU,C2P,C1C,C1P,XX,NN
107 FORMAT(9X,'POLICY',2X,'II',2X,'(AGE REPLACEMENT WITH
& MINIMAL REPAIR AND SIMPLE PM )
& ',9X,65(' '),/,
& 10X,'ALPHA = ',F3.1,17X,'MU = ',F6.1,/,

```

```

& 10X,'C2P' = $,F7.1,12X,'C1C' = $,F7.1/,
& 10X,'C1P' = $,F7.1,12X,'XX' = 'F6.1' HOURS',/
& 10X,'NO. OF SIMPLE PM' = 'NN',//
& 5X,'REPLACEMENT',6X,'COST',/ 5X,'AGE (HOURS)',5X,'
& $/HOUR' /,5X,11(' '),5X,7(' '))

DO 6 J = LL,UL,STEP
  B = REAL(J)
  X = XX
  IF ((X-B) .GE. 0) THEN
    X = B
  END IF

  DO 7 I = 0,NN-1
    SUM1 = C1C*(LAMBDA**ALPHA)*(((B+((B-X)*I))**ALPHA)
&      - (((B-X)**ALPHA)*(I**ALPHA)))
    SUM = SUM + SUM1
7  CONTINUE

  COST = (SUM + ((NN-1)*C1P) + C2P) / ((NN*B) + R2P)
  LENGTH = B*REAL(NN)
  WRITE(30,108)LENGTH,COST
108  FORMAT(5X,F6.1,5X,F9.4)
  SUM = 0.0
  6  CONTINUE

  GO TO 99

```

```

C-----
C  POLICY III (AGE REPLACEMENT - WITH TWO TYPES OF FAILURE)
C-----

```

```

300 CONTINUE

```

```

PRINT*, 'ENTER THE ESTIMATED VALUES FOR THE SHAPE
&  PARAMETER ALPHA AND THE SCALE PARAMETER MU '
READ*, ALPHA, MU
PRINT*, 'ALPHA = ', ALPHA, 'MU = ', MU

```

```

PRINT*, 'ENTER ESTIMATED VALUES FOR THE EXPECTED COST OF
&  PREVENTIVE REPLACEMENT FOLLOWED BY THE EXPECTED
&  COST OF FAILURE REPLACEMENT, AND COST OF MINIMAL
&  REPAIR'
READ*, C2P, C2C, C1C

```

```

PRINT*, 'EXPECTED COST OF A PREVENTIVE REPLACEMENT =

```

```

&                                ',C2P
&    ,EXPECTED COST OF A FAILURE REPLACEMENT  =
&                                ',C2C
&    ,EXPECTED COST OF A MINIMAL REPAIR      =
&                                ',C1C

PRINT*, 'ENTER VALUES FOR THE EXPECTED DOWNTIME OF
&    PREVENTIVE REPLACEMENT FOLLOWED BY EXPECTED
&    DOWNTIME OF A FAILURE REPLACEMENT AND DOWNTIME
&    OF MINIMAL REPAIR'
READ*, R2P, R2C, R1C
PRINT*, 'EXPECTED DOWNTIME OF A PREVENTIVE REPLACEMENT =
&                                ', R2P
&    , 'EXPECTED DOWNTIME OF A FAILURE REPLACEMENT  =
&                                ', R2C
&    , 'EXPECTED DOWNTIME OF A MINIMAL REPAIR      =
&                                ', R1C

PRINT*, 'ENTER THE VALUE FOR LOWER LIMIT, UPPER LIMIT,
&    STEP SIZE '
READ*, LL, UL, STEP
PRINT*, 'LOWER LIMIT = ', LL, 'UPPER LIMIT = ', UL, 'STEP =
&    ', STEP

PRINT*, 'ENTER VALUES FOR THE PROBABILITY OF MINIMAL
&    FAILURE AND MAJOR FAILURE'
READ*, P1, P2
PRINT*, 'THE PROBABILITY OF MINIMAL FAILURE      = 'P1,
&    AND PROBABILITY OF MAJOR FAILURE          = 'P2

PRINT*, 'ENTER THE VALUE FOR LOWER LIMIT, UPPER LIMIT,
&    STEP SIZE '
READ*, LL, UL, STEP
PRINT*, 'LOWER LIMIT = ', LL, 'UPPER LIMIT = ', UL, 'STEP =
&    ', STEP

LAMBDA = 1.0D0/MU

WRITE(30,109) ALPHA, MU, C2P, C2C, C1C, R2P, R2C, R1C, P1, P2
109 FORMAT(9X, 'POLICY', 2X, 'IIF', 2X, '(AGE REPLACEMENT WITH
& TWO TYPES OF FAILURE', 9X, 55(' '), //,
& $ 5X, 'ALPHA = ', F3.1, 11X, 'MU = ', F6.1, /,
& 5X, 'C2P = $', F7.1, 6X, 'C2C = $', F7.1, 7X, 'C1C =
& $', F7.1, /, 5X, 'R2P = ', F4.1, ' HOURS', 5X, 'R2C =
& ', F4.1, ' HOURS', 5X, 'R1C = ', F4.1, ' HOURS', /, 5X,
& 'PROBABILITY OF TYPE I FAILURE = ', F3.1, /, 5X,

```

```

& 'PROBABILITY OF TYPE II FAILURE = ',F3.1,/,
& ,5X,'REPLACEMENT',6X,'COST',10X,'APPROXIMATE',8X,
& 'EXACT',/, 5X,'AGE (HOURS)',5X,'$/HOUR',9X,'
& 'AVAILABILITY',4X,'AVAILABILITY',/,5X,11(' '),
& 4X,8(' '),8X,12(' '),      & 4X,12(' ')

```

```

DO 13 I = 0,4000,50
  B=DBLE(I)

```

```

C   CALL IMSL SUBROUTINE DQDAG

```

```

  CALL DQDAG (F,A,B,ERRABS,ERRREL,IRULE,RESULT,ERREST)

```

```

  A1 = DEXP(-((((LAMBDA*B)**ALPHA)*P1)+(B/R1C)))
  AA = A1+(RESULT*(1.0D0/R1C) )

```

```

C   OUTPUT THE AVAILABILITY AT TIME T TO FILE 13

```

```

  WRITE(13,29)B,AA
29  FORMAT(1X,F6.1,15X,F8.6)

```

```

13  CONTINUE

```

```

  REWIND (13)

```

```

  DO 43 J = 1, NDATA
    READ(13,*)XDATA(J),YDATA(J)
43  CONTINUE

```

```

  CALL DCSINT(NDATA,XDATA,YDATA,BREAK,CSCOE)

```

```

C
C           Calculate the integral of the spline
C           approximation.

```

```

DO 33 K = LL,UL,STEP
  B = DBLE(K)
  NINTV = NDATA - 1
  CALL DQDAG (G,A,B,ERRABS,ERRREL,IRULE,SOLN,ERREST)
  SF = DEXP(-(P2*((LAMBDA*B)**ALPHA)))
  CDF = 1.0D0-SF
  LENGTH = (R2P*SF)+(R2C*CDF)+SOLN
  AVAIL1 = SOLN/LENGTH

```

```

  ANSWER = DCSITG(A,B,NINTV,BREAK,CSCOE)
  AVAIL = ANSWER/LENGTH

```

```

  CALL DQDAG (H,A,B,ERRABS,ERRREL,IRULE,ANS,ERREST)

```



```

      COST = (C1C*SF*(P1*(LAMBDA*B)**ALPHA)) + (C2P*SF) +
&      (C2C*CDF) + (ALPHA*P1*P2*C1C*(LAMBDA**
&      (2.0D0*ALPHA)) *ANS)

```

```

      COSMIN = COST / LENGTH

```

```

      WRITE (80,110) B,COSMIN,AVAIL1,AVAIL
33 CONTINUE
110 FORMAT(7X,F6.1,6X,F9.5,10X,F8.6,8X,F8.6)

```

```

99 STOP
END

```

C-----

```

      DOUBLE PRECISION FUNCTION E(T)
      REAL*8 LAMBDA,ALPHA,T
      COMMON /HT/ LAMBDA,ALPHA
      E=DEXP(-((LAMBDA * T)**ALPHA))
      RETURN
      END

```

```

      DOUBLE PRECISION FUNCTION EE(T)
      REAL*8 LAMBDA,ALPHA,T,R1C,B
      COMMON /HT/ LAMBDA,ALPHA
      COMMON /FT/ R1C,B
      EE=DEXP(-(((LAMBDA * B)**ALPHA)-((LAMBDA*T)**ALPHA) +
&      (B/R1C)
&      -(T/R1C))))
      RETURN
      END

```

```

      DOUBLE PRECISION FUNCTION F(T)
      REAL*8 LAMBDA,ALPHA,T,R1C,B,P1,P2
      COMMON/FT/ R1C,B
      COMMON /HT/ LAMBDA,ALPHA
      COMMON/IT/ P1
      COMMON /GT/ P2
      F=DEXP(-(((LAMBDA * B)**ALPHA)*P1) -
&      (((LAMBDA*T)**ALPHA)*P1)+(B/R1C) -(T/R1C) +
&      (((LAMBDA*T)**ALPHA)*P2) ))
      RETURN
      END

```

```

DOUBLE PRECISION FUNCTION FF(T)
REAL*8 LAMBDA,ALPHA,T
COMMON /HT/ LAMBDA,ALPHA
FF=(T**ALPHA)*(DEXP(-(LAMBDA * T)**ALPHA)))
RETURN
END

```

```

DOUBLE PRECISION FUNCTION G(T)
REAL*8 LAMBDA,ALPHA,T,P2
COMMON/GT/ P2
COMMON /HT/ LAMBDA,ALPHA
G=DEXP(-(P2*((LAMBDA*T)**ALPHA)))
RETURN
END

```

```

DOUBLE PRECISION FUNCTION H(T)
REAL*8 LAMBDA,ALPHA,T,P2
COMMON/GT/ P2
COMMON /HT/ LAMBDA,ALPHA
H=(T**((2.0D0*ALPHA)-1.0D0)) *
& DEXP(-(P2*((LAMBDA*T)**ALPHA)))
RETURN
END

```

**POLICY 1 (AGE REPLACEMENT)**

ALPHA = 3.0	MU = 1390.0
C2P = \$25000.0	C2C = \$37500.0
R2P = 8.0 HOURS	R2C = 16.0 HOURS
MISSION DURATION = 24.0 HOURS	

<u>REPLACEMENT AGE (HOURS)</u>	<u>COST RATE (\$/HOUR)</u>	<u>AVAILABILITY (A)</u>	<u>MISSION RELIABILITY (R)</u>
900.0	32.781	0.988396	0.977947
950.0	31.900	0.988565	0.975493
1000.0	31.178	0.988681	0.972916
1050.0	30.592	0.988752	0.970214
1100.0	30.123	0.988785	0.967391
1150.0	29.754	0.988786	0.964447
1200.0	29.472	0.988761	0.961383
1250.0	29.263	0.988713	0.958200
1300.0	29.115	0.988649	0.954899
1350.0	29.020	0.988571	0.951482
1400.0	28.968	0.988483	0.947951
1450.0	28.951	0.988389	0.944306
1500.0	28.962	0.988290	0.940549
1550.0	28.995	0.988191	0.936681
1600.0	29.043	0.988093	0.932704

POLICY 1 (AGE REPLACEMENT WITH SIMPLE PREVENTIVE MAINTENANCE)

ALPHA = 3.0  
R2P = 8.0 HOURS  
R1P = 1.0 HOUR

MU = 1390.0  
R2C = 48.0 HOURS  
R = 0.1

<u>NO. OF INTERVAL</u>	<u>REPLACEMENT AGE (HOURS)</u>	<u>AVAILABILITY</u>
1	917.6	0.979421
2	1743.4	0.982197
3	2486.7	0.983305
4	3155.6	0.983591
5	3757.7	0.983585
6	4299.5	0.983480
7	4787.1	0.983344
8	5226.0	0.983206
9	5621.0	0.983075
10	5976.5	0.982955
11	6296.5	0.982848
12	6584.4	0.982751
13	6843.6	0.982664
14	7076.8	0.982587
15	7286.7	0.982518

POLICY II (AGE REPLACEMENT WITH MINIMAL REPAIR)

ALPHA = 3.0  
C2P = \$25000.0  
R2P = 8.0 HOURS

MU = 1390.0  
C1C = \$37500.0  
R1C = 1.0 HOURS

<u>REPLACEMENT AGE (HOURS)</u>	<u>COST \$/HOUR</u>	<u>APPROXIMATE AVAILABILITY</u>	<u>EXACT AVAILABILITY</u>
1200.0	40.6693	0.992845	0.992847
1300.0	42.5667	0.993258	0.993261
1400.0	44.9682	0.993593	0.993595
1500.0	47.8289	0.993862	0.993865
1600.0	51.1154	0.994076	0.994080
1700.0	54.8018	0.994245	0.994249
1800.0	58.8682	0.994374	0.994379
1900.0	63.2987	0.994469	0.994474
2000.0	68.0807	0.994532	0.994539
2100.0	73.2039	0.994569	0.994576
2200.0	78.6598	0.994581	0.994589
2300.0	84.4415	0.994571	0.994580
2400.0	90.5432	0.994540	0.994551
2500.0	96.9601	0.994490	0.994503
2600.0	103.6880	0.994423	0.994437

POLICY III (AGE REPLACEMENT WITH TWO TYPES OF FAILURE)

ALPHA = 3.0                      MU = 1390.0  
C2P = \$25000.0              C2C = \$37500.0              C1C = \$ 1000.0  
R2P = 8.0 HOURS              R2C = 24.0 HOURS              R1C = 8.0 HOURS

PROBABILITY OF TYPE I FAILURE = 0.6  
PROBABILITY OF TYPE II FAILURE = 0.4

<u>REPLACEMENT AGE (HOURS)</u>	<u>COST \$/HOUR</u>	<u>APPROXIMATE AVAILABILITY</u>	<u>EXACT AVAILABILITY</u>
1018.0	27.30690	0.989570	0.988997
1042.0	26.88558	0.989629	0.989030
1066.0	26.49157	0.989678	0.989053
1090.0	26.12318	0.989718	0.989066
1114.0	25.77884	0.989749	0.989070
1138.0	25.45711	0.989772	0.989065
1162.0	25.15666	0.989787	0.989052
1186.0	24.87625	0.989794	0.989032
1210.0	24.61474	0.989795	0.989004
1234.0	24.37107	0.989789	0.988970
1258.0	24.14424	0.989778	0.988929
1282.0	23.93333	0.989760	0.988882
1306.0	23.73747	0.989737	0.988830
1330.0	23.55583	0.989709	0.988773
1354.0	23.38767	0.989677	0.988710
1378.0	23.23225	0.989639	0.988643

# **APPENDIX B. DERIVATION OF COST RATE AND AVAILABILITY FUNCTIONS**

Let  $T$  be the time to failure

$$C = \begin{cases} C_p & T > t_p \\ C_i & T \leq t_p \end{cases}$$

$$\begin{aligned} \text{Then } E[C] &= C_p P[T > t_p] + C_i P[T \leq t_p] \\ &= C_p P[1 - F(t_p)] + C_i P[F(t_p)] \end{aligned}$$

$$L = \begin{cases} R_p + t_p & T > t_p \\ R_i + T & T \leq t_p \end{cases}$$

$$\text{Then } E[L] = E[L \mid T > t_p] P[T > t_p] + E[L \mid T \leq t_p] P[T \leq t_p]$$

Now,

$$\begin{aligned} P[T \leq t \mid T \leq t_p] &= \frac{P[T \leq t, T \leq t_p]}{P[T \leq t_p]} \\ &= \begin{cases} \frac{F(t)}{F(t_p)} & \text{if } t \leq t_p \\ 1 & \text{if } t > t_p \end{cases} \end{aligned}$$

Then,

$$E[T \mid T \leq t_p] = \int_0^{t_p} \frac{t f(t)}{F(t_p)} dt$$

Therefore

$$E[L] = (R_p + t_p) [1 - F(t_p)] + R_i [F(t_p)] + \int_0^{t_p} t f(t) dt$$

$$\begin{aligned}
&= R_p [1 - F(t_p)] + R_z [F(t_p)] + t_p [1 - F(t_p)] + \\
&\quad \int_0^{t_p} t f(t) dt \\
&= R_p [1 - F(t_p)] + R_z [F(t_p)] + t_p - t_p F(t_p) + \\
&\quad \int_0^{t_p} t f(t) dt
\end{aligned}$$

The integral  $\int_0^{t_p} t f(t) dt$  can be simplified by using

$$\int u dv = uv - \int v du$$

$$\text{then } \int_0^{t_p} t f(t) dt = t_p F(t_p) - \int_0^{t_p} F(t) dt$$

$$\text{now } E[L] = R_p [1 - F(t_p)] + R_z [F(t_p)] + t_p - \int_0^{t_p} F(t) dt$$

$$= R_p [1 - F(t_p)] + R_z [F(t_p)] + \int_0^{t_p} dt - \int_0^{t_p} F(t) dt$$

$$= R_p [1 - F(t_p)] + R_z [F(t_p)] + \int_0^{t_p} [1 - F(t)] dt$$

$$= R_p [1 - F(t_p)] + R_z [F(t_p)] + M(t_p)$$

$$\text{where } M(t_p) = \int_0^{t_p} [1 - F(t)] dt$$

$$\text{Now } G(t_p) = \frac{E[C]}{E[L]}$$

$$\begin{aligned}
&= \frac{C_p P[1 - F(t_p)] + C_z P[F(t_p)]}{R_p [1 - F(t_p)] + R_z [F(t_p)] + \int_0^{t_p} [1 - F(t)] dt}
\end{aligned}$$



Setting  $\frac{dC(t_p)}{dt_p} = 0$

we get  $C_2 f(t_p) \int_0^{t_p} [1 - F(t)] dt - C_p f(t_p) \int_0^{t_p} [1 - F(t)] dt -$

$$C_2 [F(t_p)] - C_p [F(t_p)] + C_2 [F(t_p)]^2 - C_p [F(t_p)]^2 \\ = C_p - C_p [F(t_p)] + f(t_p) [R_2 C_p - R_p C_2]$$

rearranging the terms we get

$$f(t_p) \int_0^{t_p} [1 - F(t)] dt (C_2 - C_p) - F(t_p) (1 - F(t_p)) (C_2 - C_p) =$$

$$C_p (1 - F(t_p)) + f(t_p) [R_2 C_p - R_p C_2]$$

since  $h(t_p) = \frac{f(t_p)}{1 - F(t_p)}$

then  $h(t_p) \int_0^{t_p} [1 - F(t)] dt (C_2 - C_p) - F(t_p) (C_2 - C_p) =$

$$C_p + h(t_p) [R_2 C_p - R_p C_2]$$

and this simplifies to

$$h(t_p) \int_0^{t_p} [1 - F(t)] dt - F(t_p) = \frac{C_p}{C_2 - C_p} + h(t_p) \left[ \frac{R_2 C_p - R_p C_2}{C_2 - C_p} \right]$$

Now

$$\text{Availability } A(t_p) = \frac{\text{Mean life during a cycle}}{\text{Expected length of cycle}}$$

$$= \frac{t_p [1 - F(t_p)] + \int_0^{t_p} t f(t) dt}{R_p [1 - F(t_p)] + R_f [F(t_p)] + t_p [1 - F(t_p)] + \int_0^{t_p} t f(t) dt}$$

$$= \frac{\int_0^{t_p} [1 - F(t)] dt}{R_p [1 - F(t_p)] + R_f [F(t_p)] + \int_0^{t_p} [1 - F(t)] dt}$$

Setting  $\frac{dA(t_p)}{dt} = 0$

we get {  $\int_0^{t_p} [1 - F(t)] dt + R_f [F(t_p)] + R_p [1 - F(t_p)]$  } {  $1 - F(t_p)$  }

$$= \{ \int_0^{t_p} [1 - F(t)] dt \} \{ [1 - F(t_p)] + R_f [F(t_p)] - R_p [F(t_p)] \}$$

simplifying the equation above we get,

$$\int_0^{t_p} [1 - F(t)] dt + R_f [F(t_p)] + R_p - R_p [F(t_p)] -$$

$$F(t_p) \int_0^{t_p} [1 - F(t)] dt - R_p [F(t_p)]^2 - R_p [F(t_p)] + R_p [F(t_p)]^2 =$$

$$\int_0^{t_p} [1 - F(t)] dt - F(t_p) \int_0^{t_p} [1 - F(t)] dt + \{ \int_0^{t_p} [1 - F(t)] dt \}$$

$$R_f [F(t_p)] - \{ \int_0^{t_p} [1 - F(t)] dt \} R_p [F(t_p)] \}$$

Cancelling and rearranging the terms we get

$$\{f(t_p) \left[ \int_0^{t_p} [1 - F(t)] dt \right] [R_f - R_p] \} - \{ F(t_p) [R_f - R_p - R_f F(t_p)]$$

$$+ R_p [F(t_p)] \} = R_f [1 - F(t_p)]$$

and this can be simplified to

$$\{f(t_p) \left[ \int_0^{t_p} [1 - F(t)] dt \right] [R_f - R_p] \} - \{ F(t_p) [R_f - R_p] [1 - F(t_p)] =$$

$$R_f [1 - F(t_p)]$$

Then

$$\frac{f(t_p)}{1 - F(t_p)} \int_0^{t_p} [1 - F(t)] dt - F(t_p) = \frac{R_p}{R_f - R_p}$$

$$h(t_p) \int_0^{t_p} [1 - F(t)] dt - F(t_p) = \frac{R_p}{R_f - R_p}$$

Rearranging the terms we have

$$\int_0^{t_p} [1 - F(t)] dt = \left\{ F(t_p) + \frac{R_p}{R_f - R_p} \right\} \frac{1}{h(t_p)}$$

## APPENDIX C. DESCRIPTION OF IMSL SUBROUTINE DQDAG

### **QDAG/DQDAG** (Single/Double precision)

**Purpose:** Integrate a function using a globally adaptive scheme based on Gauss-Kronrod rules.

**Usage:** CALL QDAG (F, A, B, ERRABS, ERRREL, IRULE, RESULT, ERREST)

### **Arguments**

**F** - User-supplied FUNCTION to be integrated. The form is  $F(X)$ , where

**X** - Independent variable. (Input)

**F** - The function value. (Output)

F must be declared EXTERNAL in the calling program.

**A** - Lower limit of integration. (Input)

**B** - Upper limit of integration. (Input)

**ERRABS** - Absolute accuracy desired. (Input)

**ERRREL** - Relative accuracy desired. (Input)

**IRULE** - Choice of quadrature rule. (Input)

A Gauss-Kronrod rule is used with

7 - 15 points if IRULE = 1

10 - 21 points if IRULE = 2

15 - 31 points if IRULE = 3

20 - 41 points if IRULE = 4

25 - 51 points if IRULE = 5

30 - 61 points if IRULE = 6

IRULE = 2 is recommended for most functions.

If the function has a peak singularity use IRULE = 1

If the function is oscillatory use IRULE = 6

**RESULT** - Estimate of the integral from A to B of F. (Output)

**ERREST** - Estimate of the absolute value of the error. (Output)

### **Notes**

QDAG is a general-purpose integrator that uses a globally adaptive scheme in order to reduce the absolute error. It subdivides the interval  $[A,B]$  and uses a  $(2k + 1)$ -point Gauss-Kronrod rule to estimate the integral over each subinterval. The error for each subinterval is estimated by comparison with the  $k$ -point Gauss quadrature rule. The subinterval with the largest estimated error is then bisected and the same procedure is applied to both halves. The bisection process is continued until either the error criterion is satisfied, roundoff error is detected, the subintervals become too small, or the maximum number of subintervals allowed is reached.

## **APPENDIX D. DESCRIPTION OF IMSL SUBROUTINE DCSINT**

**CSINT/DCSINT**      (Single/Double precision)

**Purpose:**            Compute the cubic spline interpolant .

**Usage:**            CALL DCSINT (NDATA, XDATA, YDATA, BREAK, CSCOEf)

### **Arguments**

**NDATA** -      Number of data points. (input)

                NDATA must be at least 2.

**XDATA** -      Array of length NDATA containing the data point abscissas.  
                (input)

**YDATA** -      Array of length NDATA containing the data point ordinates.  
                (input)

**BREAK** -      Array of length NDATA containing the breakpoints for the  
                piecewise cubic representation. (Output)

**CSCOEf** -      Matrix of size 4 by NDATA containing the local coefficients of  
                the cubic pieces. (Output)

### **Notes**

DCSINT computes the second derivative cubic spline interpolant to a set of data points  $(x_i, y_i)$  for  $i = 1, 2, \dots, \text{NDATA} = N$ . The breakpoints of the spline are the

abscissas. Endpoint conditions are automatically determined by the program. These conditions correspond to the "not-a-knot" condition, which requires that the third derivative of the spline be continuous at the second and next-to-last breakpoint.

## APPENDIX E. DESCRIPTION OF IMSL FUNCTION DCSITG

**CSITG/DCSITG** (Single/Double precision)

**Purpose:** Evaluate the integral of a cubic spline

**Usage:** CSITG(A, B, NINTV, BREAK, CSCOEFF)

### Argument

- A** - Lower limit of integration. (Input)
- B** - Upper limit of integration. (Output)
- NINTV** - Number of polynomial pieces. (Input)
- BREAK** - Array of length NINTV+1 containing the breakpoints for the piecewise cubic representation. (Input)
- CSCOEFF** - Matrix of size 4 by NINTV+1 containing the local coefficients of the cubic pieces. (Input)
- DCSITG** - Value of the integral of the spline from A to B. (Output)

### Notes

DCSITG evaluates the integral of a cubic spline over an interval. A cubic spline is a piecewise polynomial of order 4.



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